Distribuzioni

Binomiale

$$P(x) = \frac{N!}{x!(N-x)!} p^x (1-p)^{N-x}$$
$$\langle x \rangle = Np$$
$$\langle x^2 \rangle - \langle x \rangle^2 = Np(1-p)$$

Normale

$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$
$$\langle x \rangle = \mu$$
$$\langle x^2 \rangle - \langle x \rangle^2 = \sigma^2$$

Poissoniana

$$P(x) = \frac{\lambda^x}{k!} e^{-\lambda}$$
$$\lambda = \frac{p}{n}$$
$$\langle x \rangle = \lambda$$
$$\langle x^2 \rangle - \langle x \rangle^2 = \lambda$$

Integrali

$$\int_0^{+\infty} dx e^{-ax^2} x = \frac{1}{2a} \qquad \int_{-\infty}^{+\infty} dx e^{-ax^2+bx} = \sqrt{\frac{\pi}{a}} e^{\frac{b^2}{4a}} \qquad \mathrm{erf}(s) = \frac{2}{\sqrt{\pi}} \int_0^s dx\ e^{-x^2}$$

$$\Gamma(s) = \int_0^{\infty} dx x^{s-1} e^{-x} \qquad \zeta(s) = \frac{1}{\Gamma(s)} \int_0^{+\infty} dx \frac{x^{s-1}}{e^x - 1} = \frac{1}{\Gamma(s)(1-2^{1-s})} \int_0^{+\infty} dx \frac{x^{s-1}}{e^x + 1}$$

Termodinamica

$$dE = TdS - PdV + \mu dN \qquad A = E - TS \qquad H = E + PV \qquad G = A + PV = \mu N$$

$$T = \left(\frac{\partial E}{\partial S}\right)_{N,V} \qquad P = -\left(\frac{\partial A}{\partial V}\right)_{N,T} \qquad \mu = \left(\frac{\partial A}{\partial N}\right)_{V,T} \qquad S = -\left(\frac{\partial A}{\partial T}\right)_{N,V}$$

$$c_V = \frac{1}{N}\left(\frac{\partial E}{\partial T}\right)_{N,V} \qquad c_P = \frac{1}{N}\left(\frac{\partial H}{\partial T}\right)_{N,P}$$

Gas Perfetto Classico

$$Z = \frac{Z_1^N}{N!} \qquad Z_1 = \int \frac{d\vec{q}d\vec{p}}{h^d} e^{-\beta H(\vec{q},\vec{p})} \qquad \langle E \rangle = -N \frac{\partial \ln Z_1}{\partial \beta} \qquad A = -\frac{N}{\beta} \ln\left(\frac{Z_1 e}{N}\right)$$

$$G(\epsilon) = \int \frac{d\vec{q}d\vec{p}}{h^d} \delta(\epsilon - H(\vec{q},\vec{p})) \qquad P(\epsilon) = \frac{1}{Z_1} G(\epsilon) e^{-\beta\epsilon}$$

Gas perfetto quantistico

$$\langle N \rangle = \int_{-\infty}^{\infty} d\epsilon\ G(\epsilon) n(\epsilon) \qquad \langle E \rangle = \int_{-\infty}^{\infty} d\epsilon\ G(\epsilon) n(\epsilon)\ \epsilon \qquad PV\beta = \pm \int_{-\infty}^{+\infty} d\epsilon\ G(\epsilon) \ln(1 \pm n(\epsilon))$$

$$G(\epsilon) = g_s \int \frac{d\vec{q}d\vec{p}}{h^d} \delta(\epsilon - H(\vec{q},\vec{p})) \qquad n(\epsilon) = (e^{\beta(\epsilon-\mu)} \pm 1)^{-1} \qquad \begin{cases} + \to fermioni \\ - \to bosoni \end{cases}$$