Superconductivity in low-dimensional systems

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- Lecture 1: Superconductivity and coherence: the old part of the story
- Lecture 2: SC in low (actually 2) dimensions.
  What's up with SC? Breaking news and old revisited stuff

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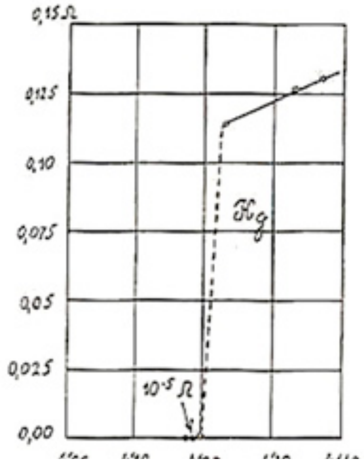
Outline of lecture1: SC in a nutshell

- phenomenology;
- London theory and rigidity;
- BCS theory: the standard model;
- Ginzburg-Landau theory;
- SC and coherence;
- Josephson effect.

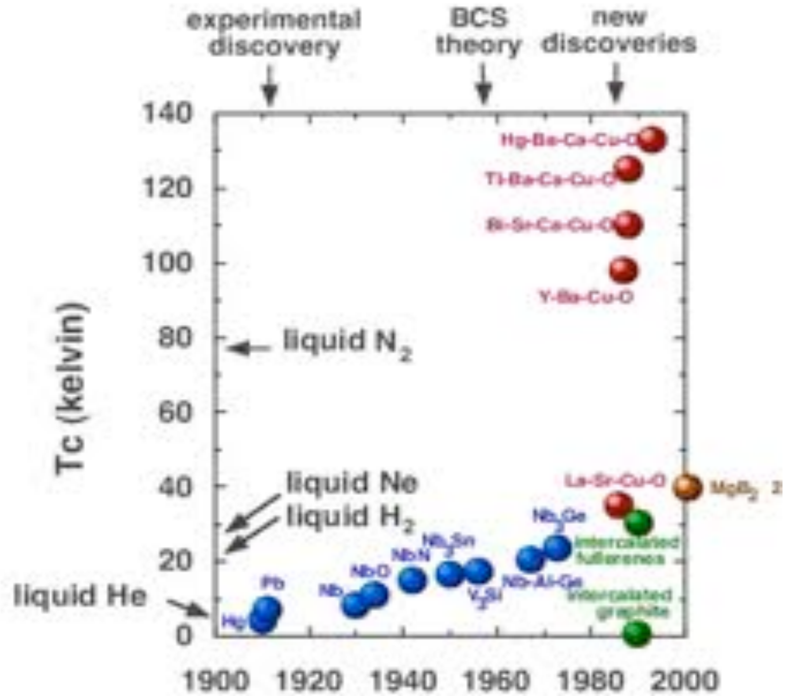
Aim: What is the SC state? Why do we discuss it in connection to coherence?

Some basic SC phenomenology:

1) the resistance vanishes below some critical T_c



April 1911 G. Holst (in Onnes' Lab in Leiden first observes vanishing resistance in Hg)



J. Bednorz, K.A. Müller
Z. Phys. B 1986
high-Tc SC

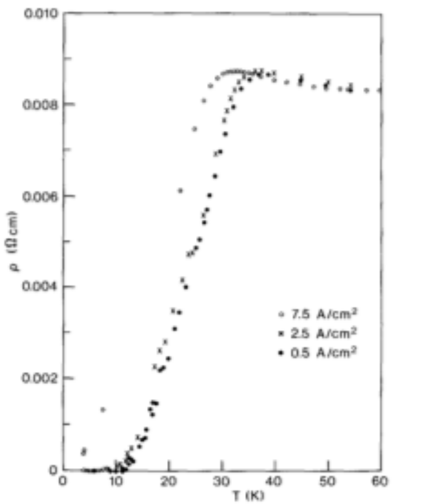
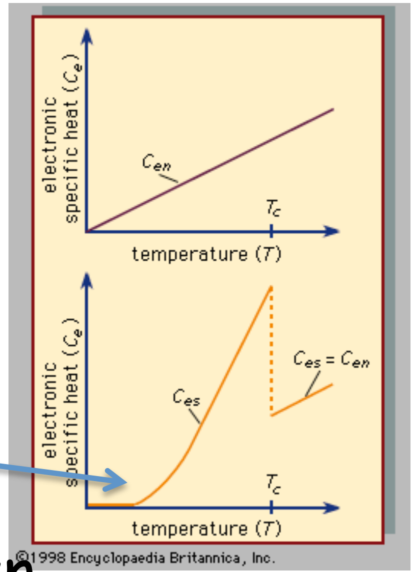
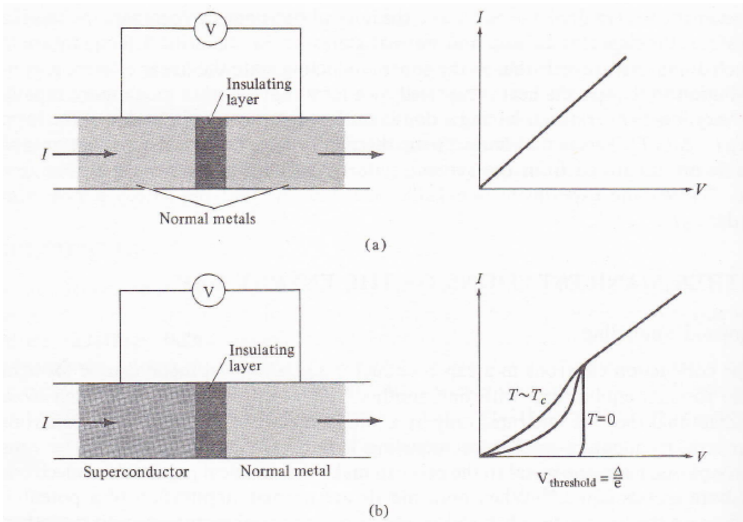


Fig. 3. Low-temperature resistivity of a sample with $x(\mathrm{Ba}) = 0.75$, recorded for different current densities

2) Specific heat:
jump at Tc ⇒ true phase transition
(2^{nd} order) occurs at Tc;
exponential decrease indicates a gap
in electronic excitations
ubiquitous in traditional SCs, but now
many examples of gapless SC are known
Entropy: tends to zero below Tc:
what kind of order establishes below Tc?

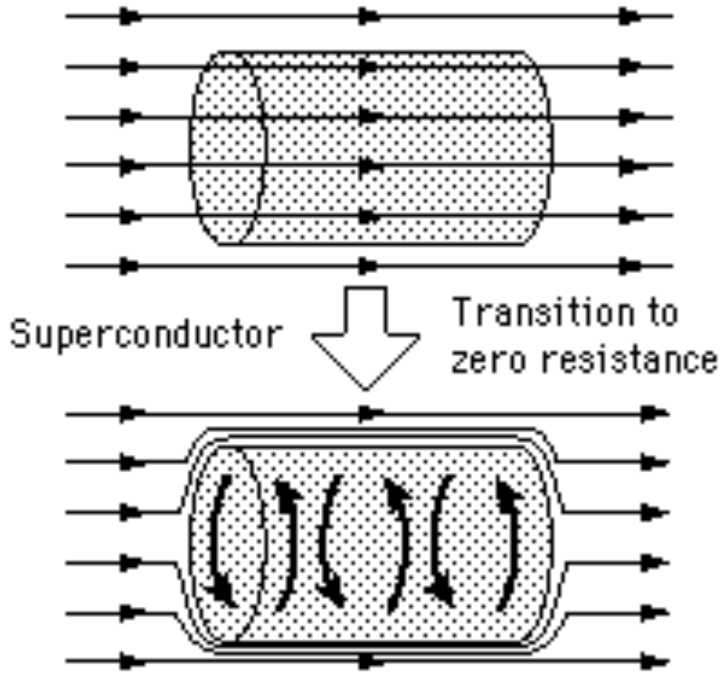


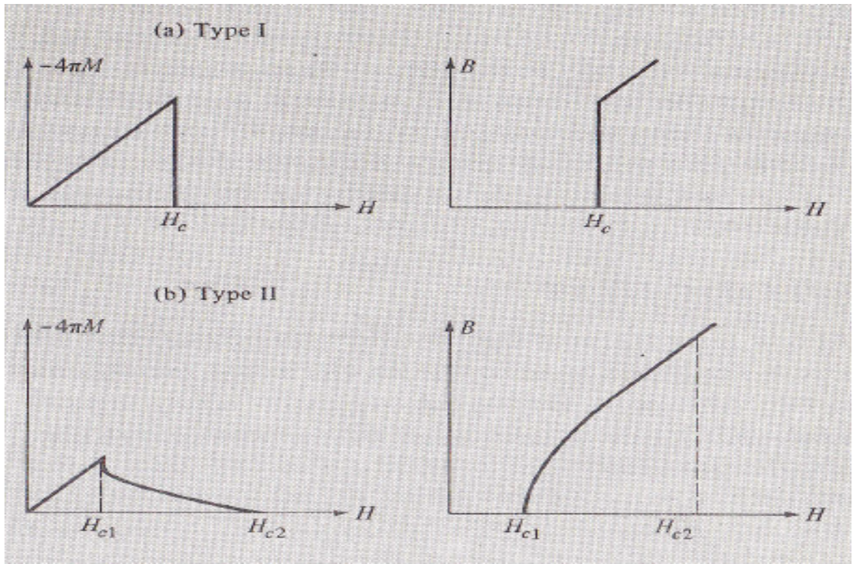
3) Tunnelling:
again a gap is found at
$T \leq T_c$



4) Meissner effect (1937):
a sheet of supercurrents completely screens the magnetic field inside the SC
B=0

This is the distinctive feature w.r.t.a perfect conductor ...





Two distinct behaviors under magnetic field:
Type I: only complete Meissner effect
Type II: Meissner effect complete up to Hc1 and then only partial up to Hc2

London's theory

See, e.g., J. R. Schrieffer "Theory of superconductivity" p. 10
Two-fluid picture: a mixture of normal and superfluid electrons

i) $\frac{\partial J_s}{\partial t} = \frac{n_s e^2}{m} E;$ (F=ma for superfluid electrons)

ii) $J_n = \sigma_n E;$ (Ohm's law for normal electrons, negligible for transport)

iii) $\nabla \times E = -\frac{1}{c}\frac{\partial B}{\partial t}$ Maxwell equation

i) and iii) imply the eq. for a perfect conductor

$$\frac{d}{dt}\left(\nabla \times J_s + \frac{n_s e^2}{mc} B \right) = 0$$

Constant

London phenomenologically assumes const=0

$$\nabla \times J_s + \frac{n_s e^2}{mc} B = 0$$

London's equation

London's equation implies Meissner effect and B=0 inside the SC for any history...

$$\nabla \times B = \frac{4\pi}{c} J_s$$

$$\nabla \times \nabla \times B = \frac{4\pi}{c} \nabla \times J_s = -\left(\frac{4\pi n_s e^2}{mc^2}\right) B$$

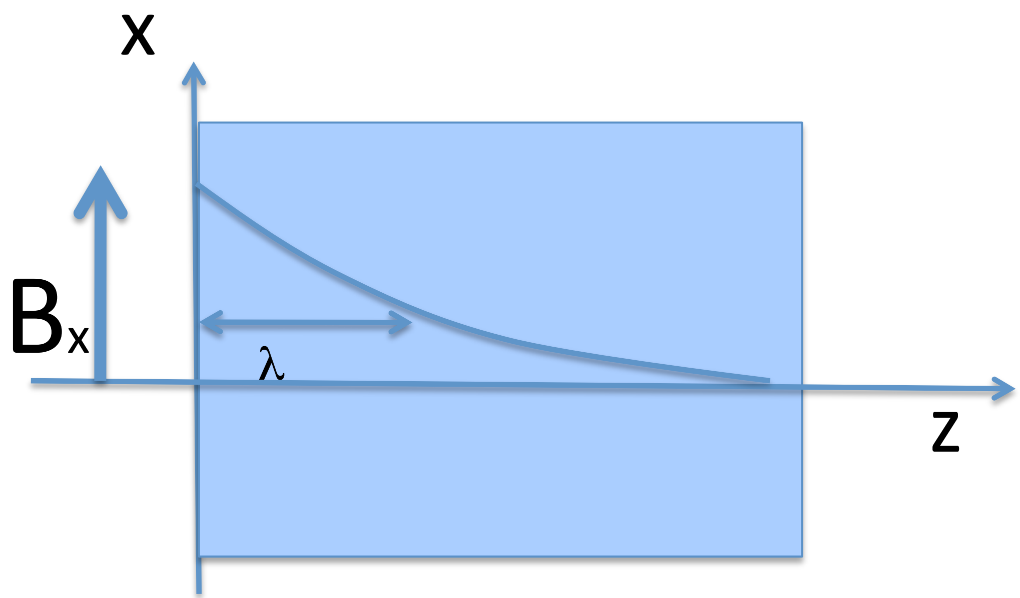
$$\frac{1}{\lambda^2}$$

Maxwell equation (neglect J_n and displacement currents for slowly varying fields)

λ is the London penetration depth

This implies Meissner effect. For instance for a flat vacuum-SC interface one finds

$$B_x(z) = B_x(0) e^{-\frac{z}{\lambda}}$$



$$\nabla \times J_s + \frac{n_s e^2}{mc} B = \nabla \times J_s + \frac{n_s e^2}{mc} \nabla \times A = 0$$

Suitably choosing the (London) gauge $\nabla \bullet A = 0$ and boundary conditions (like, for a simply connected isolated SC, $A_\perp = 0$ one can equivalently rewrite L. eq. as

$$J_s = -\frac{n_s e^2}{mc} A$$

Only seemingly non gauge invariant, but A is uniquely determined by the London gauge+boundary conditions

Rigidity of the SC wavefunction

$$mv_s = p - \frac{e}{c}A \Rightarrow \langle J_s \rangle = n_s e \langle v_s \rangle = ne\left(\langle p \rangle_A - \frac{e}{c}A\right)$$

is the sum of the paramagnetic and diamagnetic currents, where the paramagnetic current is given by

$$n_s e \langle p \rangle_A \equiv \langle J_p(r) \rangle = \frac{e\hbar}{2mi}\sum_{j=1}^{N_s}\int dr_1 \ldots dr_{N_s}\left[\psi_s^* \nabla_j \psi_s - \left(\nabla_j \psi_s^*\right)\psi_s\right]\delta\left(r - r_j\right)$$

while the diamagnetic current is $\langle J_d(r) \rangle = -\frac{e^2}{mc} n_s(r) A(r).$

London's hypotesis: the SC wavefunction does not change by perturbing the systems with a transverse slowly varying vector potential (rigidity):

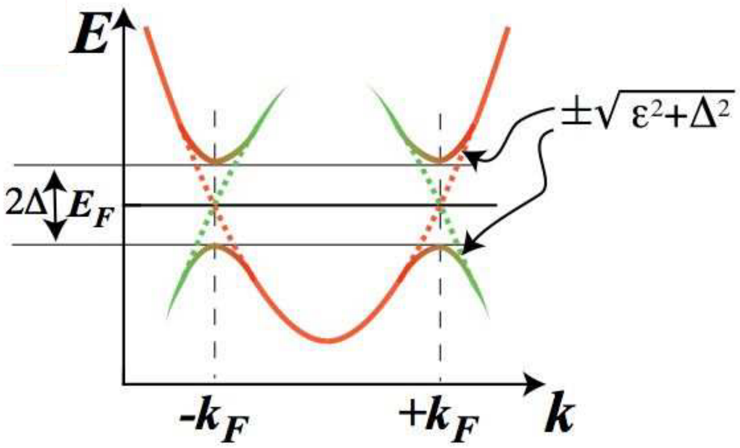
$$\psi_s(A \neq 0) = \psi_s(A = 0)$$

Since $\langle p \rangle_{A=0} = 0$ rigidity implies $\langle p \rangle_A = 0 \Rightarrow \langle J_p \rangle = 0 \Rightarrow \langle J_s \rangle = \langle J_d \rangle = -\frac{ne^2}{mc}A$

Again London's equation....Meissner effect and infinite d.c. conductivity

$\langle J_s(\omega) \rangle = -\frac{ne^2}{mc}\frac{1}{i\omega}\left(i\omega A(\omega)\right) = \sigma(\omega) E(\omega)$ with $\sigma(\omega) = \pi\delta(\omega) + i\frac{P}{\omega}$

Coherence Length



How many k states are substantially readjusted to form the new (paired) condensed state?

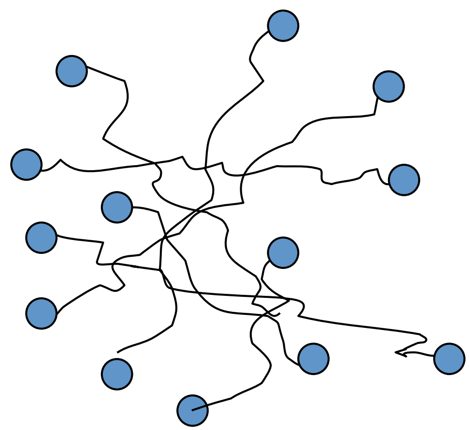
$$2\Delta \approx v_F \delta k$$

To form pairs the electron states are modified on length scales

$$\xi_0 \sim \delta k^{-1} \sim \frac{\hbar v_F}{\pi \Delta}$$

ξ_0 is the typical size of an electron pair >>a

Many interconnected pairs form a rigid condensed state

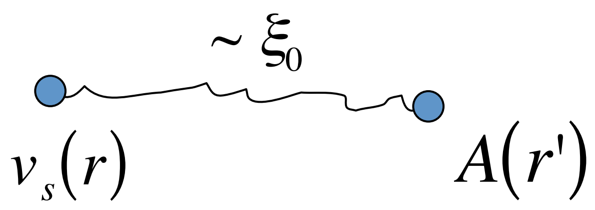


Also the electromagnetic response feels this length scale (Pippard) with a non-local

response $$\nabla \times \mathbf{j}(\mathbf{r}) = -\int d\mathbf{r}' K(\mathbf{r}-\mathbf{r}')\mathbf{B}(\mathbf{r}').$$

$$J_s(r) = C\int dr' K(r-r')A(r')$$

The e.m. response kernel K lives on a range $\sim \xi_0$: the electronic current in r feels the field in r' due to the electron-electron correlations due to pairing



Notice: if the field A(r') varies on length scales $\lambda >> \xi_0$ then the local London relation $J_s = -\frac{n_s e^2}{mc} A$ is recovered.

Bardeen, Cooper, Schrieffer theory: The standard model

Low Tc (standard metals)

Normal state:

High energy e-e repulsion ($\sim E_F \sim 10$eV)+ fast screening processes

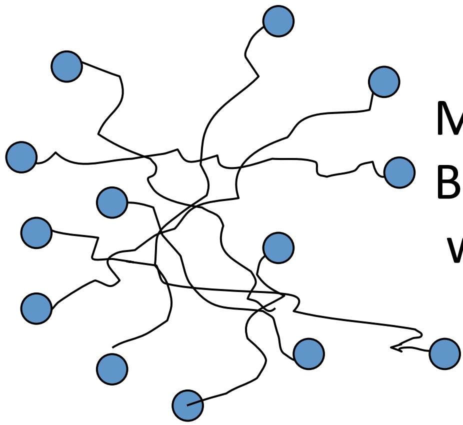
IR asymptotic freedom →

Nearly free electron gas at low energy (Landau QP's)

Superconducting state

Attraction (pairing) at low energy from phonons (small expansion parameter, Migdal $\frac{\omega_D}{E_F} << 1$)

Large pair size $\xi_0 >> a$: weak phase flucts. and rigid w.f.



Mean-field BCS works well

All these concepts are challenged in current research on (low-D) SC

The BCS ground state

$$\Psi(r_1,\ldots,r_{2N}) = Antisymm\left[\varphi(r_1 - r_2)\varphi(r_3 - r_4)\ldots\varphi(r_{2N-1} - r_{2N})\right] \equiv \Psi(N)$$

working with fixed N pairs is cumbersome (more on this later on). BCS introduce a trial WF

$$\left|\psi_\phi\right\rangle = \mathrm{N}\prod_k \left(1 + g_k e^{i\phi} c^+_{k\uparrow} c^+_{-k\downarrow}\right)\left|0\right\rangle$$

Normalization factor $\mathrm{N} = \dfrac{1}{\sqrt{\prod\left(1+\left|g_k\right|^2\right)}}$

$|\psi\rangle$ is a coherent superposition of states with 0,2,4,... electrons (actually Landau QPs) paired into singlets (so called Cooper pairs). The g's are determined by the minimization of the reduced hamiltonian $H_{red} = \sum_{k\sigma}\varepsilon_k n_{k\sigma} + \sum_{kk'} V_{kk'} b^+_k b_{k'}$ with $b^+_k = c^+_{k\uparrow} c^+_{-k\downarrow}$ pair creation operator

$$\delta\langle W\rangle = \left\langle\psi_\phi\right|\left(H_{red} - \mu\hat{N}\right)\left|\psi_\phi\right\rangle = 0$$

$$\left\langle\psi_\phi\right|\hat{N}\left|\psi_\phi\right\rangle = \langle N\rangle \quad \sqrt{\langle N^2\rangle - \langle N\rangle^2} \sim \sqrt{\langle N\rangle}$$

N is not fixed, but it fluctuate little w.r.t. <N>

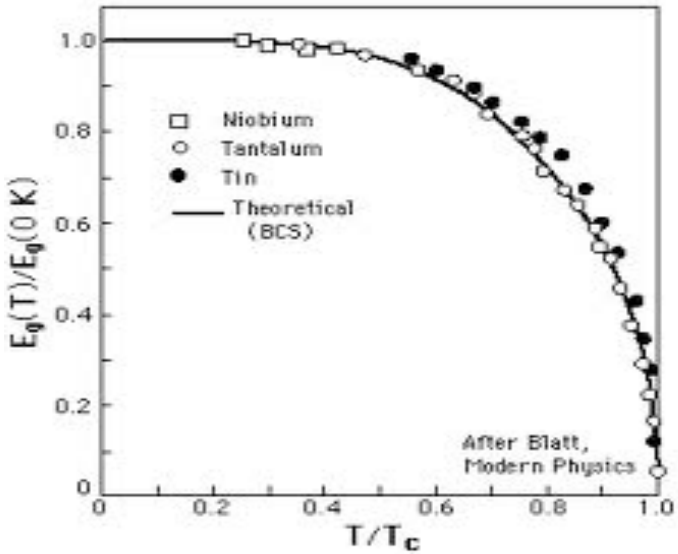
For $V_{kk'} = -V < 0$ (in an energy shell $|\xi_k - \mu| < \omega_D$) one finds a spectrum with a gap

$$E_k = \sqrt{(\varepsilon_k - \mu)^2 + |\Delta|^2}$$

$$\Delta \approx 2\omega_D e^{-\frac{1}{\nu(0)V}}$$ v(0) is the DOS at the Fermi level

$$K_B T_c \approx 1.14\omega_D e^{-\frac{1}{\nu(0)V}}$$

Δ and Tc have the same (non analytic) form and are the same energy scale



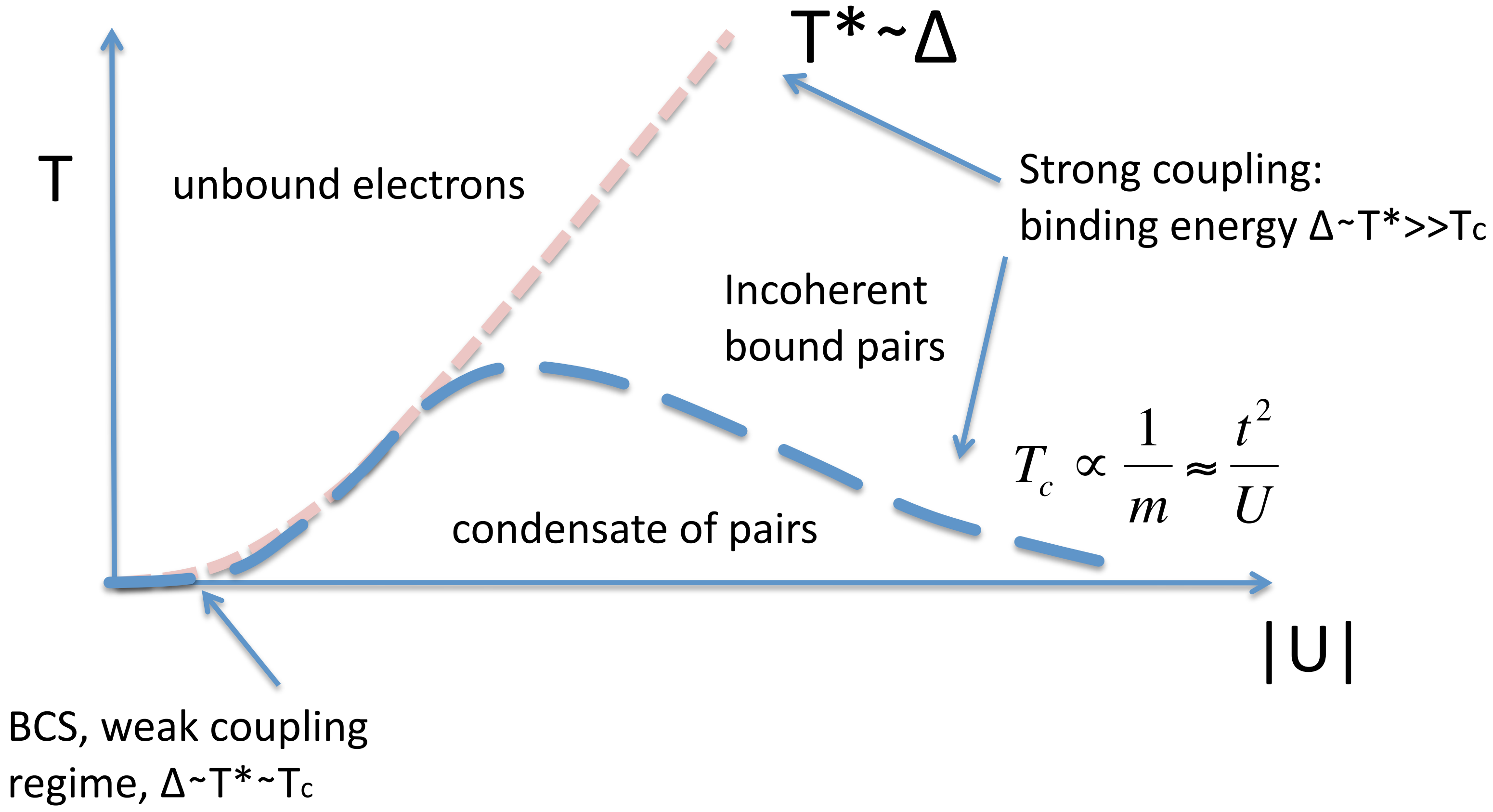
This is a distinctive feature of BCS SC: as soon as the gap opens a coherent SC state occurs

A counterexample: the Negative U Hubbard model

$$H_{Hub} = -t \sum_{\langle ij \rangle \sigma} c^{+}_{i\sigma} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} \qquad U < 0$$

Kinetic term: its FT gives the tight-binding band

Local attraction: it gives pairing



In which sense "coherent"?

$|\psi_\phi\rangle$ is labeled by the global phase ϕ

$|\Psi_N\rangle$ and $|\psi_\phi\rangle$ are complete basis sets (quite similarly to eigenstates of r and p) and one can go from one basis to the other

$$|\Psi_N\rangle = \frac{1}{2\pi}\int_0^{2\pi} d\phi \cdot e^{iN\phi}|\psi_\phi\rangle; \quad \Longleftrightarrow \quad |\psi_\phi\rangle = \sum_N e^{-i\phi N}|\Psi_N\rangle$$

$[N,\phi] = i$ with the total pair-N and glob. phase operators $N = i\partial_\phi, \phi = -i\partial_N$

$$|\psi_\phi\rangle = \mathrm{N}\prod_k\left(1 + g_k e^{i\phi} c^+_{k\uparrow} c^+_{-k\downarrow}\right)|0\rangle \propto \exp\left(e^{i\phi}\sum_k g_k b_k^+\right) \equiv \exp\left(e^{i\phi} b^+\right)$$

Similar to a Glauber coherent state

$$b^+ \equiv \sum_k g_k b_k^+,$$

$$\left[b_k, b_{k'}^+\right] = \delta_{k,k'}\left[1 - \left(n_{k\uparrow} + n_{k\downarrow}\right)\right]$$

$$\left[b_k, b_{k'}\right] = \left[b_k^+, b_{k'}^+\right] = 0$$

the b's are not really boson operators: their algebra is the same as that of Pauli spin operators (P.W. Anderson (1958))

Using the field operators $\psi_\sigma^+(x) = \sum_k e^{ik\cdot x} c_{k\sigma}^+$ one can show (see [Schrieffer]) that

$$\left|\Psi_N\right\rangle = \int_0^{2\pi} d\phi \cdot e^{iN\phi}\left|\psi_\phi\right\rangle = \frac{\mathrm{N}}{N!}\left(\sum_k g_k b_k^+\right)^N \left|0\right\rangle = \frac{\mathrm{N}}{N!}\left[\int\int dx dx' \psi_\uparrow^+(x)\psi_\downarrow^+(x')\left(\sum_k g_k e^{ik\cdot(x-x')}\right)\right]^N$$

$$= Antisymm\left[\varphi(x_1 - x_2)\ldots\varphi(x_{2N-1} - x_{2N})\right]$$

with $\varphi(x - x')$ the WF of the Cooper pairs

N pairs in the same state φ: similar to the Bose condensation. However, here the pairs are not really bosons....(large overlaps on scale ξ_0 and the same energy scale for the electron binding energy and the condensation temperature T_c)
(Counterexample: the negative-U Hubbard model)

The system undergoes a phase transition from incoherent unpaired electrons to a condensate of coherent pairs.

$$|\psi_N\rangle = \int_0^{2\pi} d\phi\, e^{-iN\phi}\, |\psi_\phi\rangle = \int_0^{2\pi} d\phi\, e^{-iN\phi}\, \mathcal{N}\, e^{e^{i\phi}\sum_k g_k b_k^+}\, |0\rangle =$$

$$= \mathcal{N}\int_0^{2\pi} d\phi\, e^{-iN\phi} \sum_m \frac{e^{i\phi m}}{m!} \Big(\underbrace{\sum_k g_k b_k^+}_{b^+}\Big)^m |0\rangle = \sum_m \frac{1}{m!} (b^+)^m \int_0^{2\pi} d\phi\, e^{i\phi(m-N)} |0\rangle$$

$$= \mathcal{N}\frac{1}{N!} \Big(\sum_k g_k b_k^+\Big)^N |0\rangle = \frac{\mathcal{N}}{N!}\Big(\sum_k g_k \int dx\, e^{ikx}\, \psi_\uparrow^+(x) \int dx'\, e^{-ikx'} \psi_\downarrow^+(x')\Big)^N$$

$$= \frac{\mathcal{N}}{N!}\Big[\int dx\, dx'\, \psi_\uparrow^+(x)\, \psi_\downarrow^+(x') \underbrace{\sum_k g_k e^{ik(x-x')}}_{\varphi(x-x')}\Big]^N$$

The Ginzburg-Landau theory

[de Gennes],[Schrieffer],[NageleOrland]

$\left|\psi_\phi\right\rangle$ is a coherent state. Therefore it has overlap with itself even if one destroys a pair

$$\left\langle\psi_\phi\right|\psi_\downarrow(x)\psi_\uparrow(x)\left|\psi_\phi\right\rangle \equiv \Psi(x)$$

Gorkov 1958

has the meaning of the pair WF with x center of mass and ξ_0 small with respect to the typical distances over which Ψ varies.

This can be taken as a complex order parameter

$$\Psi(x) = 0, \quad T > T_c$$
$$\Psi(x) \neq 0, \quad T < T_c$$

To obtain an analytic free energy F (2nd-order transition) the GL functional is analytic and given by an expansion in terms of Ψ(r)

$$F[\Psi] = F_n + \int d^D r \left[\alpha\left|\Psi(r)\right|^2 + \frac{\beta}{2}\left|\Psi(r)^4\right| + \frac{1}{2m^*}\left|\left(-i\hbar\nabla - \frac{e^*}{c}A\right)\Psi(r)\right|^2 + \frac{h^2}{8\pi} \right]$$

with $\alpha(T) = \alpha'(T - T_c), \quad \beta > 0$

the stationarity conditions yield the GL equations

$$\alpha\Psi + \frac{\beta}{2}|\Psi|^2\Psi + \frac{1}{2m^*}\left(-i\hbar\nabla - \frac{e^*}{c}A\right)^2\Psi = 0$$

$$J = \frac{e^*\hbar}{2im^*}\left(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*\right) - \frac{e^{*2}}{m^*c^2}|\Psi|^2 A$$

taking $\Psi(r) = |\Psi(r)|e^{i\varphi(r)}$ the 2nd eq. becomes:

$$J(r) = \frac{e^*}{m^*}|\Psi(r)|^2\hbar\left(\nabla\varphi(r) - \frac{e^*}{\hbar c}A(r)\right)$$

taking $\nabla\times$ on both sides one gets London eq. provided

$$|\Psi(r)|^2 \approx |\Psi_0|^2 = n_s^{pairs} = \frac{n_s}{2},$$

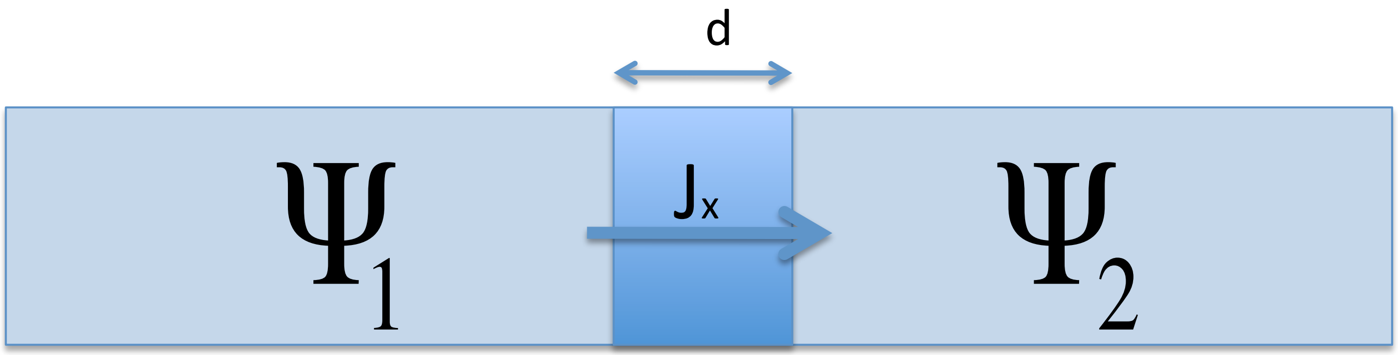
$$e^* = 2e, \quad m^* = 2m$$

$$\frac{\hbar}{m}\left(\nabla\varphi(r) - \frac{2e}{\hbar c}A(r)\right) = v_s$$

is the gauge-invariant velocity

The Josephson effect

[Anderson],[Tinkham,Ch. 6],[deGennes]



$$J_x = \frac{2e\hbar}{m^*}|\Psi|^2\left(\frac{\partial\varphi}{\partial x} - \frac{2\pi}{\Phi_0}A_x\right) \xrightarrow{\left(d\to 0,\,|\Psi|^2\to 0\right)} J_x \approx \frac{1}{d}\int_0^d dx J_x = \underbrace{\frac{2e\hbar}{m^*}\frac{|\Psi|^2}{d}}_{J_0}\gamma$$

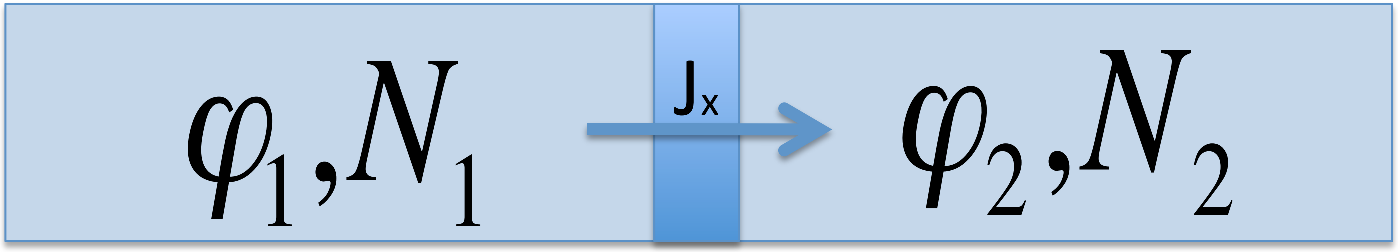
Where $\Phi_0 = \frac{hc}{2e}$ is the flux quantum

where the g.i. phase difference $\gamma \equiv \varphi_2 - \varphi_1 - \frac{2\pi}{\Phi_0}\int_1^2 dx A_x$

Actually the phase difference may only be defined modulo 2π. Then $J_x \equiv J_0 \sin\gamma$

$J_0 = \frac{\pi\Delta}{2eR_n} \quad (T = 0)$ Anderson

$J_0 = \frac{\pi\Delta(T)}{2eR_n}\tanh\left(\frac{\Delta(T)}{2K_B T}\right) \quad (T > 0)$ Ambegaokar, Baratoff



the two nearby SCs 1 and 2 one have two conjugate variables:
the g.i. phase difference γ and the numbers of electron pairs N1 and N2
What is their dynamics?

$$I = 2e\frac{dN_2}{dt} = -2e\frac{dN_1}{dt}$$

Remember that $\left[N,\varphi\right] = i \Rightarrow \Delta N \Delta\varphi \geq 1$ uncertainty relation (quite similar to that for r and p...).However, since $\langle N \rangle \sim 10^{20}$, one can satisfy $\Delta N / \langle N \rangle \sim \Delta\varphi \sim 10^{-10}$

One can adopt a semiclassical view in which both N and φ are fixed with precision $1/10^{10}$ and use a Hamilton classical mechanics analogy:

$$r \leftrightarrow N$$

$$p \leftrightarrow \varphi$$

$$H \leftrightarrow F$$

Free energy instead of H at finite T....

$$F_{12}(\gamma) = const - \frac{\hbar I_0}{2e}\cos\gamma$$

(because it must be periodic in γ and even because the free energy cannot change by replacing ψ and ψ*)

$$i)\quad \frac{d\gamma}{dt} = \frac{d\varphi_2}{dt} - \frac{d\varphi_1}{dt} = -2\frac{\partial F}{\partial N} = \frac{2}{\hbar}\left(\mu_1 - \mu_2\right)$$

If there is a difference in electrochemical potential due to a bias potential V, then the phase oscillates with frequency

$$\omega_J = \frac{2eV}{\hbar}$$

Josephson frequency relation:
a d.c. V produces an oscillating current

$$J(t) = J_0 \sin\left[\gamma_0 + \frac{2eV}{\hbar}t\right]$$

$$ii)\quad \frac{I_{12}}{2e} = \frac{dN_2^*}{dt} = -\frac{dN_1^*}{dt} = -\frac{1}{\hbar}\frac{\partial F_{12}}{\partial \varphi_1} = \frac{1}{\hbar}\frac{\partial F_{12}}{\partial \varphi_2} = \frac{1}{\hbar}\frac{\partial F_{12}}{\partial \gamma}$$

If one assumes $F_{12}(\gamma) = const - \frac{\hbar I_0}{2e}\cos\gamma$ one finds again:

$$I_{12}(t) = I_0 \sin\left[\gamma(t)\right] \qquad [I_0 = J_0(Area)]$$

Summary: The traditional "dogmas"

•Pairs arise because (strong) e-e repulsion is screened at low energy and (weak) phonons dominate;

•Pairs are large and interconnected;

•The SC state is a ***rigid*** condensate of Cooper pairs (not really bosons);

•Tc and Δ are the same energy scale;

•At Tc a (coherent) state forms with a given global phase;

Next lecture will mostly be on how "dogmas" are violated

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SC, insulators and metals

$$\psi_s(A) = \psi_s(0) + \sum_\alpha \frac{\langle \psi_\alpha | H' | \psi_s \rangle}{E_\alpha - E_s} + \dots$$

$[H' \propto p \cdot A]$ perturbation

$J_s = KA$ with the response kernel $K \propto \sum_\alpha \left| \frac{\langle \psi_\alpha | H' | \psi_s \rangle}{E_\alpha - E_s} \right|^2 + \frac{1}{\lambda^2} \equiv R + \frac{1}{\lambda^2}$

•If $\psi_s(A)$ is rigid: $\psi_s(A) = \psi_s(0) \Rightarrow \langle \psi_\alpha | H' | \psi_s \rangle = 0$

•If a gap is present $E_\alpha - E_s > 0$ the denominators in R do not vanish

Both conditions are realized in traditional SCs: R(q→0)→0 only the diamagnetic term remains and one recovers London Eq.

Remarks:

i) the gap alone is not enough for SC: insulators have a gap, but the w.f. is not rigid and $\langle \psi_\alpha | H' | \psi_s \rangle \neq 0$ then

$$R(q \to 0) \to -\frac{1}{\lambda^2} + O(\omega) \Rightarrow K \to 0 + O(\omega)$$ Ohm's law follows

ii) if there is no gap:

- normal metal $\langle \psi_\alpha | H' | \psi_s \rangle \to 0;\quad (E_\alpha - E_s) \to 0 \quad R(q \to 0) \to -\frac{1}{\lambda^2} + O(\omega) \Rightarrow K \to 0 + O(\omega)$
- gapless SC $\langle \psi_\alpha | H' | \psi_s \rangle \to 0; (E_\alpha - E_s) \to 0$ but $R(q \to 0) \neq -\frac{1}{\lambda^2} + O(\omega) \Rightarrow K \to const$

The Anderson-Higgs mechanism

P. W. Anderson 1958, P. W. Anderson 1963, P. Higgs 1964
[NegeleOrland],[Okun]

$$F[\Phi] = F_n + \int d^D r \left[\alpha |\Phi(r)|^2 + \frac{\beta}{2} |\Phi(r)^4| + \frac{1}{2m^*} \left| \left(-i\hbar\nabla - \frac{e^*}{c} A \right) \Phi(r) \right|^2 + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]$$

$$\phi(x) = e^{i\xi(x)} \left(\phi_0 + \eta(x)\right)$$
$$\phi^*(x) = e^{-i\xi(x)} \left(\phi_0 + \eta(x)\right)$$

$$\phi(x) \to e^{-i\theta(x)} \phi(x)$$
$$\phi^*(x) \to e^{i\theta(x)} \phi^*(x)$$
$$A^\mu(x) \to A^\mu(x) + \frac{1}{e} \partial^\mu \theta(x)$$

gauge transformation

If one choses the gauge, $\xi(r) = \vartheta(r)$ then:
$\langle \xi\xi \rangle$ flucts (which would give the Goldstone boson in the long wavelength) are gauged away;

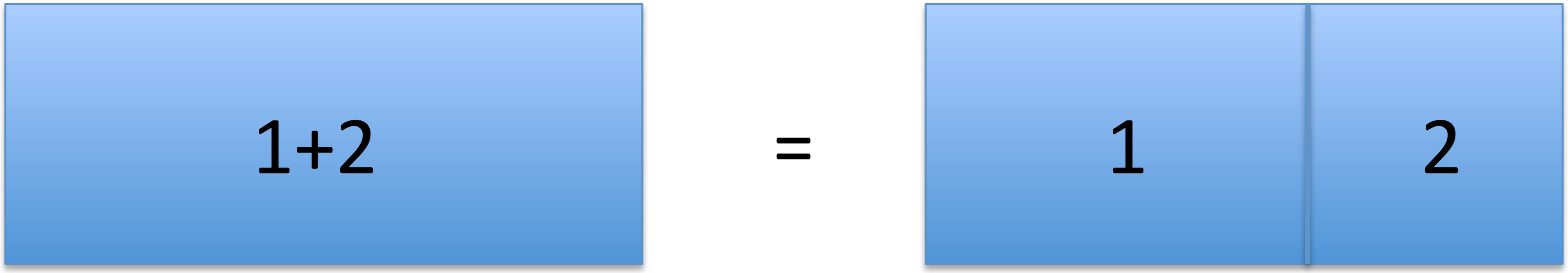
$$\phi(x) \to e^{-i\xi(x)} \phi(x) = e^{-i\xi(x)} \left[e^{i\xi(x)} \left(\phi_0 + \eta(x)\right) \right] = \phi_0 + \eta(x)$$
$$\phi^*(x) \to e^{i\xi(x)} \phi^*(x) = e^{i\xi(x)} \left[e^{-i\xi(x)} \left(\phi_0 + \eta(x)\right) \right] = \phi_0 + \eta(x)$$
$$A^\mu(x) \to A^\mu(x) + \frac{1}{e} \partial^\mu \xi(x) = \tilde{A}^\mu(x) \; .$$

$$\mathcal{L} = (\partial^\mu - ie\tilde{A}^\mu)(\phi_0+\eta)(\partial_\mu + ie\tilde{A}_\mu)(\phi_0+\eta) - r_0(\phi_0+\eta)^2 - \frac{u_0}{2}(\phi_0+\eta)^4 - \frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}$$

	Field	Modes	Number of Components	m^2
Non Interacting Fields	Scalar	η	1	$-2r_0$
		ξ	1	0
	em	A^ν	2 (transverse)	0
Interacting Fields Symmetric Phase ($r_0 > 0$)	Scalar	ϕ, ϕ^*	2	r_0
	em	A^μ	2 (transverse)	0
Interacting Fields Broken symmetry phase ($r_0 < 0$)	Scalar	η	1	$-2r_0$
	em	$\tilde{A}^\mu$	3	$-2e^2r_0/u_0$

Table 4.2 Summary of the degrees of freedom for a charged scalar field and electromagnetic field.

Meissner effect



$$\left|\psi_{\phi}^{1+2}\right\rangle = \left|\psi_{\phi}^{1}\right\rangle \otimes \left|\psi_{\phi}^{2}\right\rangle$$

If we insist to work with $\left|\psi_{N}^{i}\right\rangle = \frac{1}{2\pi}\int_{0}^{2\pi} d\phi\, e^{-iN_i\phi}\left|\psi_{\phi}^{i}\right\rangle \quad (i = 1,2,1+2)$ then

$$\left|\psi_{N_{1+2}}^{1+2}\right\rangle \neq \left|\psi_{N_1}^{1}\right\rangle \otimes \left|\psi_{N_2}^{2}\right\rangle$$

Now we look at the Ginzburg-Landau theory.