Superconductivity in low-dimensional systems

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Lecture 2: SC in low (actually 2) dimensions. What's up with SC? Breaking news and old revisited stuff

Aim: show how the learned concepts and dogmas are used/challenged in present research

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Why (quasi) 2D superconductors are interesting and "hot"?

•Layered or 2D systems are relatively easy to manipulate:

-they can be built and designed with various techniques (MBE, etching, ...) like Josephson-Junction arrays, thin films, ...

-electron density can be varied in various ways [interlayer doping (e.g. high-Tc cuprates), field-effect (oxide interfaces like LAO/STO, example of E-tuned SC),...];

•Competing mechanisms in 2D are weaker (like AF-spin order and charge-ordering in cuprates) or stronger (like disorder in films and interfaces)

•Interesting objects for fundamental physics:

-role of fluctuations in low D, physics of vortices, Berezinski-Kosterliz-Thouless (BKT) transition,...

-SC properties can be tuned (and e.g. Tc $\rightarrow$0) to study the SC-Insulator or SC-Metal quantum phase transition

-different ways disorder and interactions can affect SC ...

Short premise: quantum phase transitions (quick survey) [Sondhi]

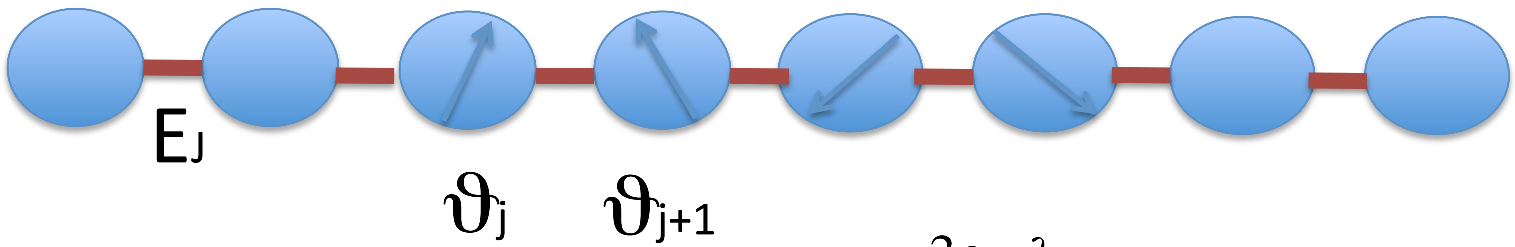
$Z(\beta) = \sum_n \langle n | e^{-\beta H} | n \rangle$ The density-matrix operator $e^{-\beta H}$ similar to time evolution $e^{-iH\tau/\hbar}$ provided $\tau = -i\hbar\beta$

Z takes the form of a sum of imaginary-time transition amplitudes for the system to start in some state $|n\rangle$ and return to the same state after an imaginary time interval $-i\hbar\beta$. Thus we see that calculating the thermodynamics of a quantum system is the same as calculating transition amplitudes for its evolution in imaginary time, with the total time interval fixed by the temperature of interest.

The Feynman path integral describing the (imaginary) time evolution of each degree of freedom becomes equivalent to the partition function in D+1 dimensions

A simple example: the 1D Josephson-Junction array

Δ >>T : in each grain electrons are well paired, but L~ξ and the phase can fluctuate



$$H = \frac{C}{2}\sum_j V_j^2 - E_J \cos\left(\hat{\vartheta}_j - \hat{\vartheta}_{j+1}\right)$$

where $V_j \equiv -i\frac{2e}{C}\frac{\partial}{\partial \vartheta_j}$ is the voltage on the j-th junction naturally related to the number of Cooper pairs on grain j (remember that $n_j = -i\frac{\partial}{\partial \vartheta_j}$

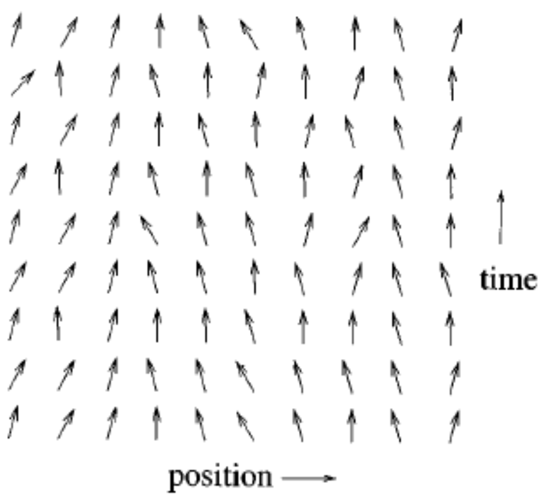


FIG. 2. Typical path or time history of a 1D Josephson-junction array. Note that this is equivalent to one of the configurations of a 1+1D classical XY model. The long-range correlations shown here are typical of the superconducting phase of the 1D array or, equivalently, of the ordered phase of the classical model.

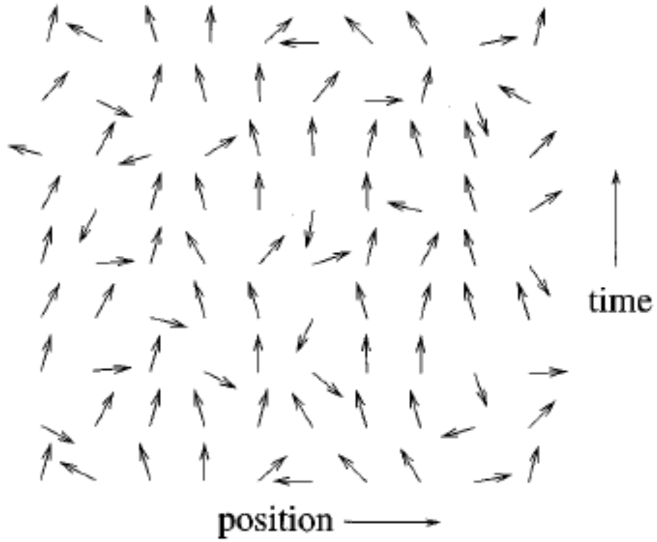


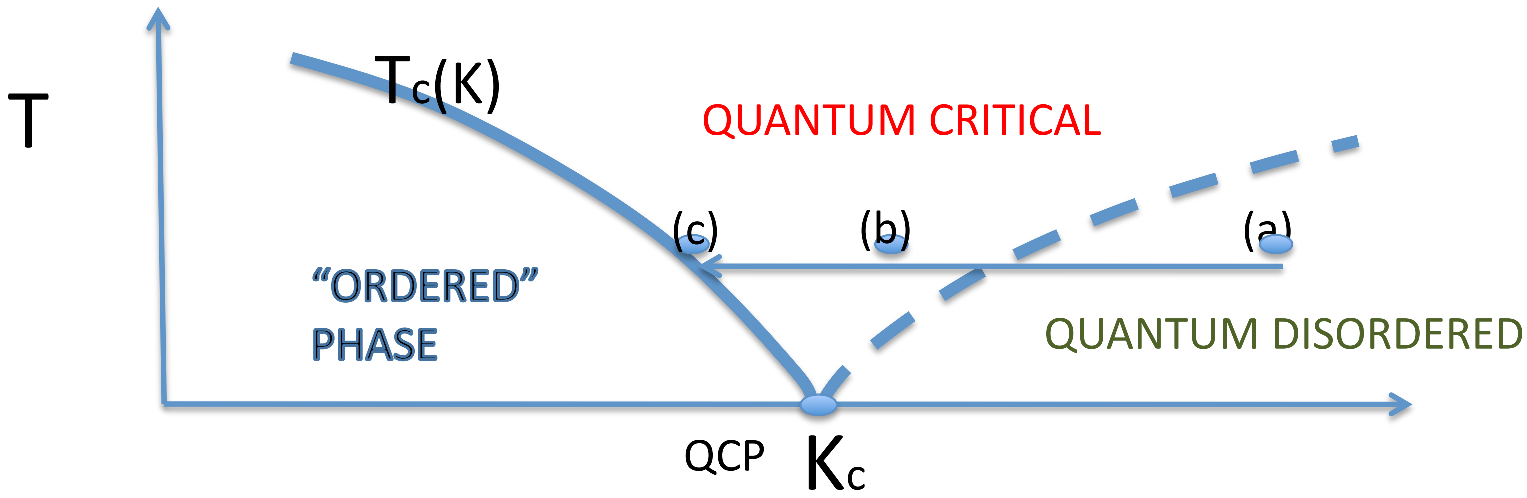
FIG. 3. Typical path or time history of a 1D Josephson-junction array in the insulating phase, where correlations fall off exponentially in both space and time. This corresponds to the disordered phase in the classical model.

The sum of all the imaginary-time evolutions to get Z(β) is equivalent to the partition function in 1+1 dimensions with

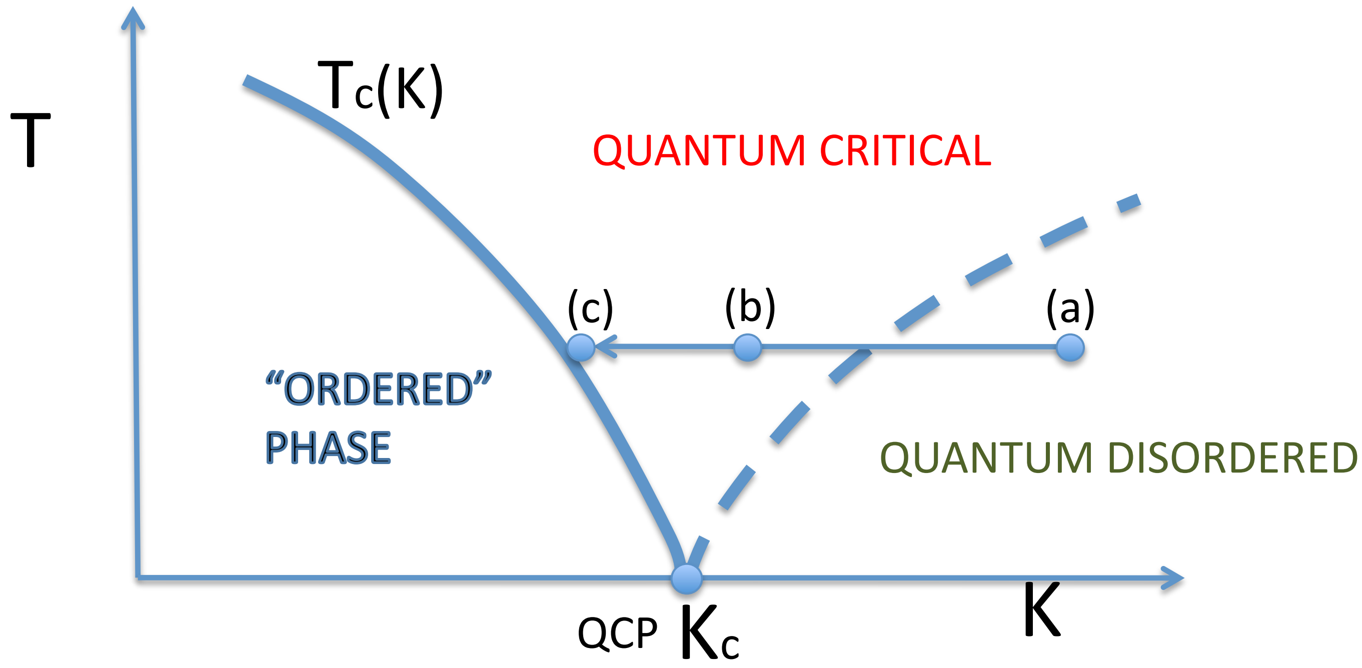
$$H_{XY} = \frac{1}{K}\sum_{\langle ij\rangle}\cos\left(\vartheta_i - \vartheta_j\right)$$

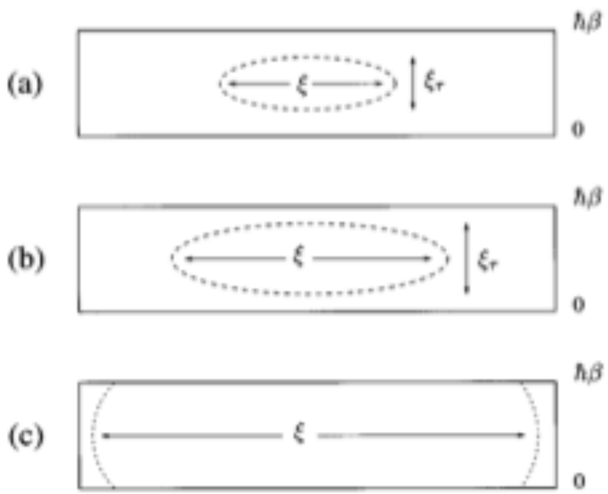
where $K \approx \sqrt{E_C / E_J}$ with $E_C = \frac{(2e)^2}{C}$ being the capacitive charging energy:

(small K: small E_C or large E_J) ⇒ charge can fluctuate a lot and phases can be more "rigid"
(large K: largel E_C or small E_J) ⇒ charge cannot fluctuate a lot and phases are more loosely bound between the sites. Thus
Small K: ordered phases, large K: disordered phase
1+1 dimension can be generalized to D+1 dimensions.



Some more “pills” of quantum criticality





At T=0 both ξ(K) and ξ_τ(K) diverge for $\delta \equiv (K-K_c) \rightarrow 0$ as

$$\xi \sim |\delta|^{-\nu}, \xi_\tau \sim \xi^z$$ with *z* dynamical critical index

$$\xi_\tau^{-1} \sim \xi^{-z} \sim \frac{1}{\tau_{fl}} \sim \omega_{fl}$$ define the typical energy scale of flucts.

At finite T, in the QD region flucts. are mostly quantum

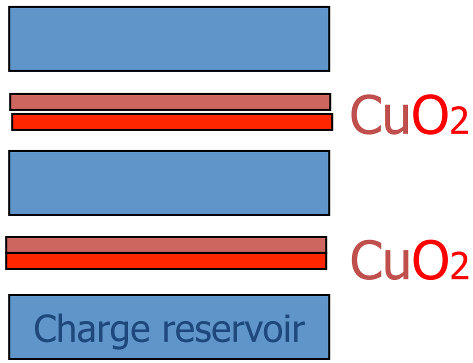
$$\xi \sim |\delta|^{-\nu}$$

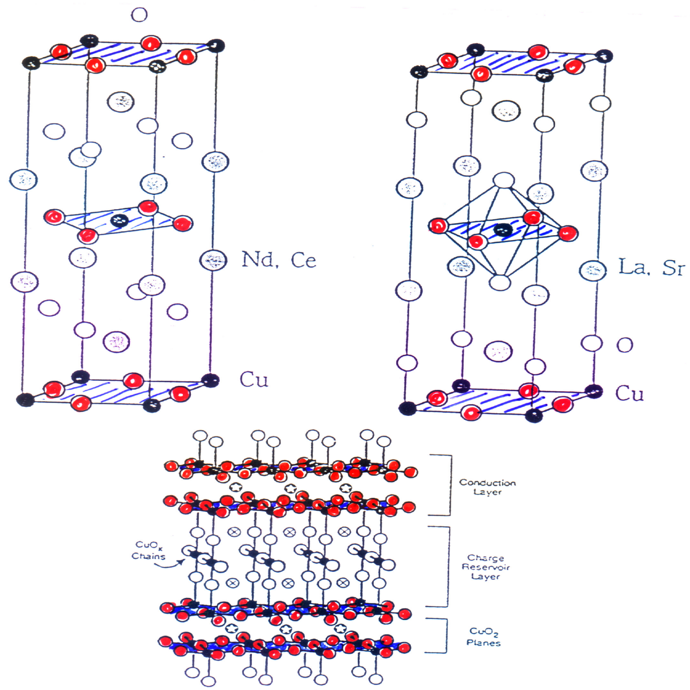
In the QC region flucts. are both thermal and quantum $\xi \sim T^{-\frac{1}{z}}$

T is the most important parameter ruling the physics

A first “lab” of quasi 2D superconductors: high-Tc cuprates

J. Bednorz, K.A. Müller, 1986, ...+26 years of frantic activity.....





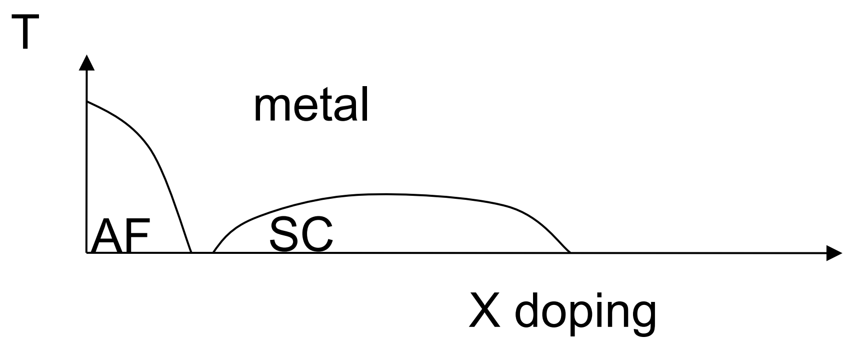
1) Weakly coupled CuO_2 planar structures

Strong anisotropy → quasi-2D systems

2) When one hole per CuO_2 cell is present (half-filling) the cuprates are insulating (and AF)

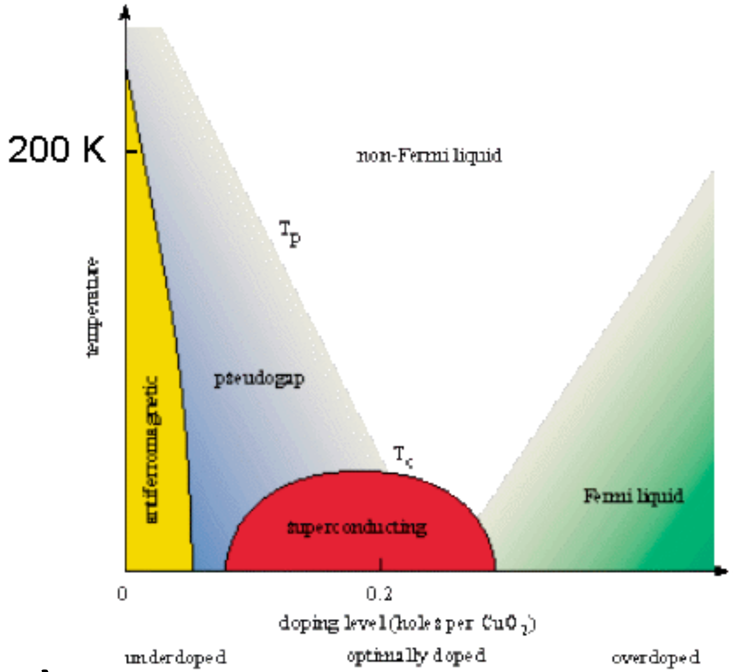
↓

e-e interactions are strong

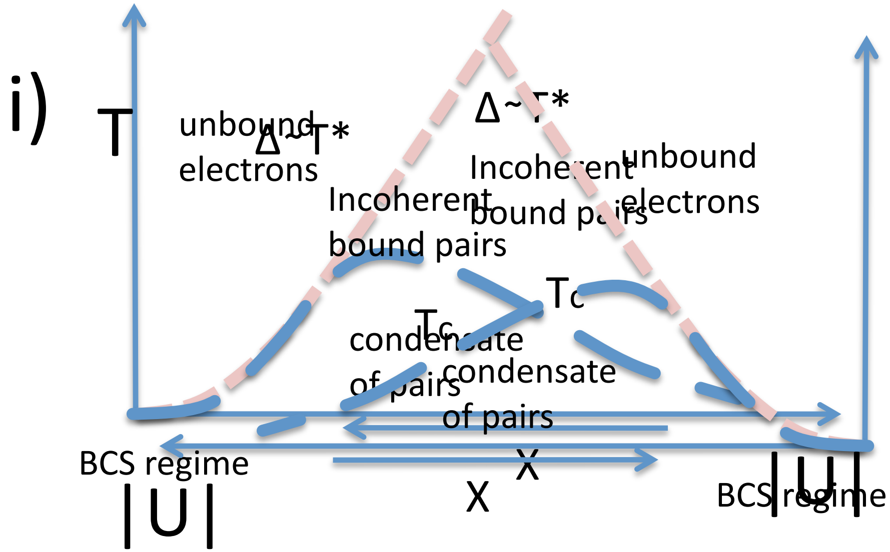


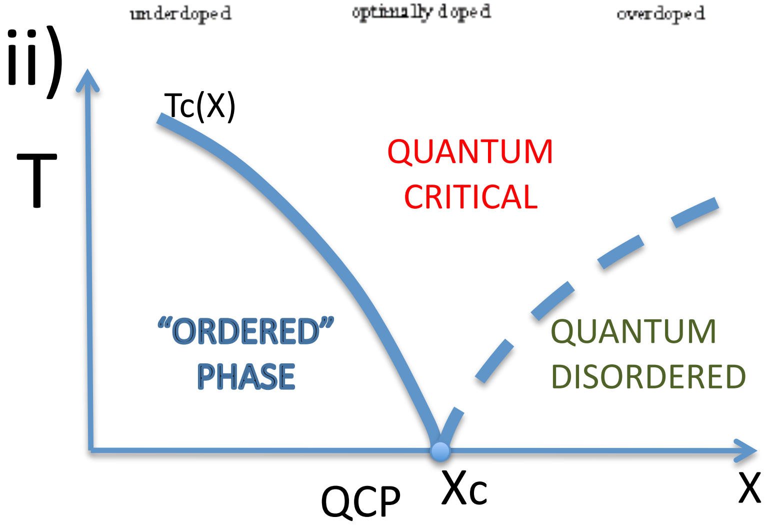
What are the “hot” physical issues in cuprates?

Cuprate phase diagram



Remember the negative-U Hubbard model?





Two distinct lines of thought: In the PG phase

i) there are only incoherent bound pairs

(anomalies due to strong e-e correlations and pairs

ii) there is a “hidden” ordered phase and a QCP underneath the SC dome around optimal doping Xc

Low dimensionality weakens the “ordered phase” and leaves dynamical quantum fluctuations (source of pairing?)

Short list of hot SC-related issues in cuprates

•What is the "normal" state: FL of Landau QPs or anomalous Non-FL state?

•Superconducting mechanism;

•Physics of SC transition: BCS or preformed pair condensation?

•role of phase fluctuations: any signature of BKT transition? Can vortex-antivortex pairs affect the normal-state properties?

•Quantum criticality:

-Is there any competing phase (and related dynamical low-energy modes)?

-SC-Insulator transition: role of disorder, polaronic effects, inhomogeneities, competing phases,....

Cuprates are a big "cocktail" of several "revolutionary" issues : all the "dogmas" of the BCS paradigm are challenged

High Tc cuprates

Normal state:

Strong interactions (both repulsive and attractive)+ low dimensionality+ small E_F

What happens at low energy? FL or NFL?

Just preformed pairs below T* or there is a competing phase (like, e.g., charge-ordered) with low-energy critical dynamical flucts.?

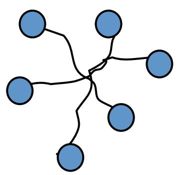
In this case how they interact with the QPs?

Superconducting state

Attraction and repulsion of the same order (non-phononic pairing?)

Strong pairing $\Rightarrow$ small pair size

$\xi_0 \approx 3 - 5a$



Mean-field is questionable

Large phase flucts.

Likely occurrence of preformed incoherent pairs below T*

Low Tc (standard metals)

Normal state:

High energy e-e repulsion (~E_F~10eV)+ fast screening processes

Nearly free electron gas at low energy (Landau QP's)

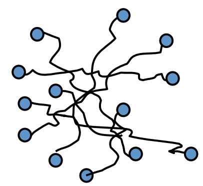
Superconducting state

Attraction (pairing) at low energy from phonons

(small expansion parameter, Migdal $\frac{\omega_D}{E_F} << 1$)

Large pair size $\xi_0 >> a$

weak phase flucts. and rigid w.f.

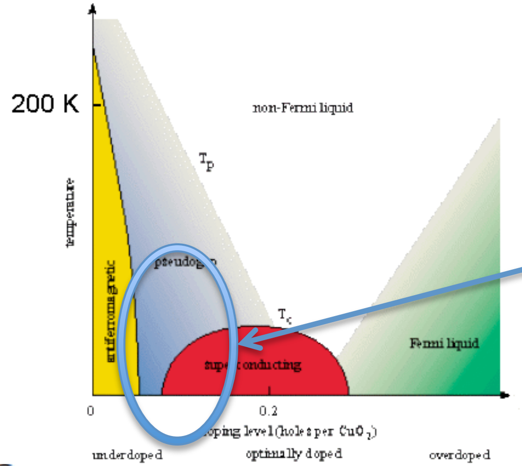


All the very basic ingredients of BCS are questioned in cuprates....

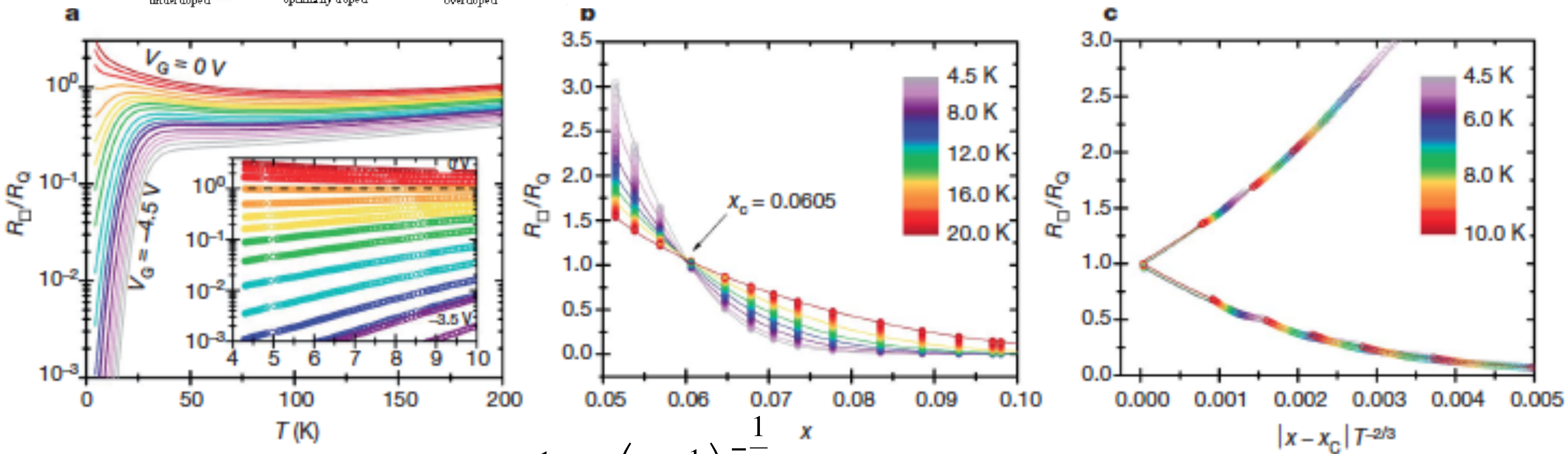
SC-Insulator transition in strongly underdoped cuprates

Bollinger et al., Nature 2011. electric-field driven SC

Cuprate phase diagram



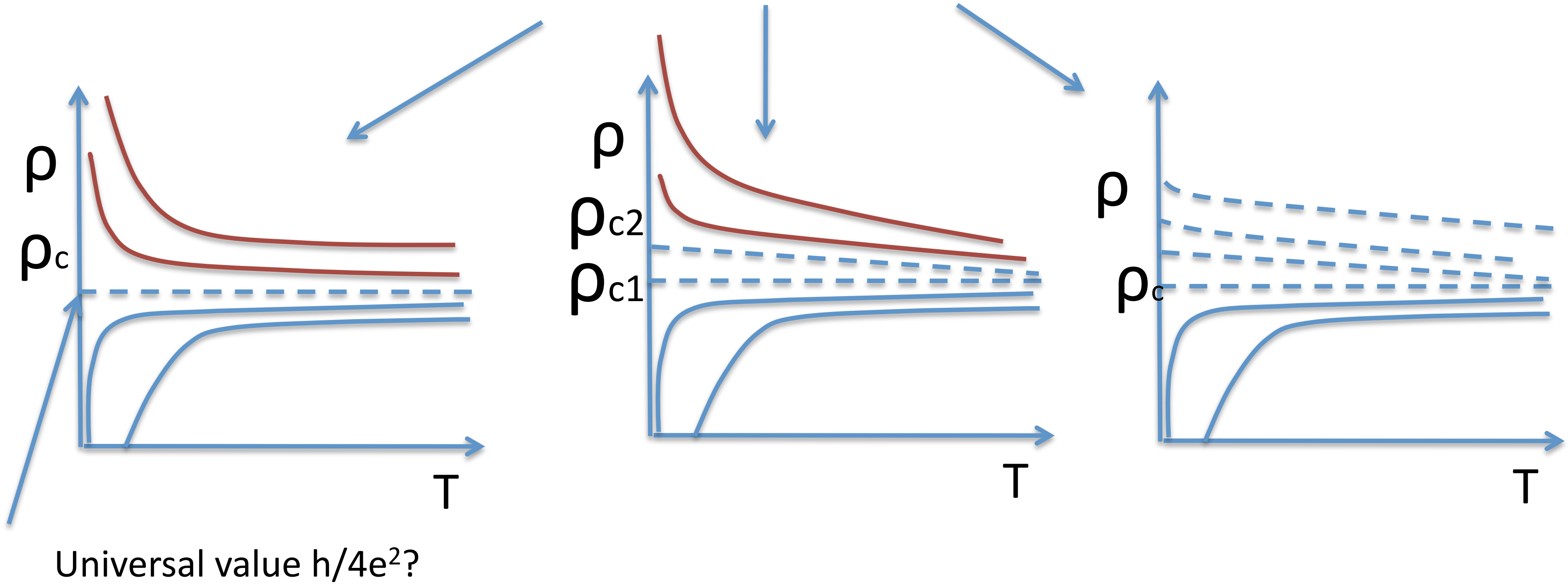
The doping is set near the SIT and then the hole density is changed by V_G



$$|x - x_c| \sim \xi^{-\frac{1}{\nu}} \sim \left(T^{-\frac{1}{z}}\right)^{-\frac{1}{\nu}}$$

$u = |x - x_c| T^{-1/z\nu}$, with $z\nu = 1.5$.

How many SC-I, SC-M-I, SC-M transitions?



The type of transition, the universality class, and so on depend on:

- The morphology of the system (homogenus, granular, honeycomb,....)

- The way the transition is induced (film width, amount of disorder, magnetic field, electron density by doping or gating,....)

How many SC-I, SC-M, SC-M-I transitions?

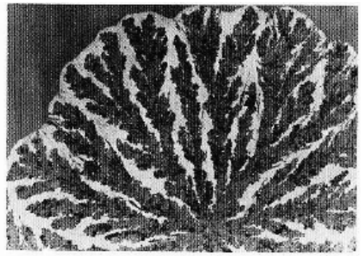
1. Percolation: a “geometrical” effect

2. Granular superconductors: R-shunted JJA:
   What matters are phase fluctuations, and transport due to fermionic QPs

3. The fermionic scenario: SC dies because pairing is spoiled

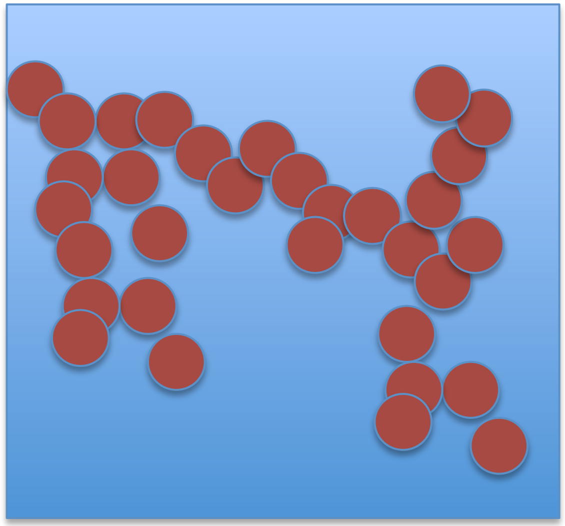
4. the Bosonic scenarios:Fisher & Co., Feigel’man,Ioffe,Mezard,Kravtsov,...SC dies, but fermions remain paired.

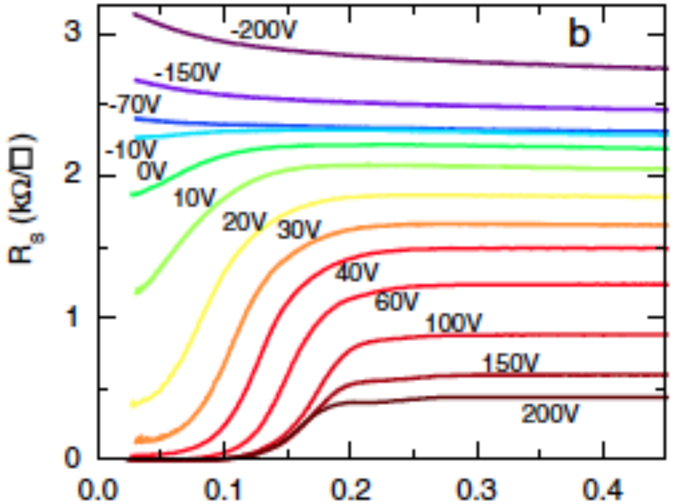
1. Percolation



Pieces of coherent SC add to a coherent cluster
(e.g. by pouring more charge into the system). When the growing cluster crosses the whole sample, global SC sets in....

SC-I or SC-M depend on the matrix embedding the SC cluster





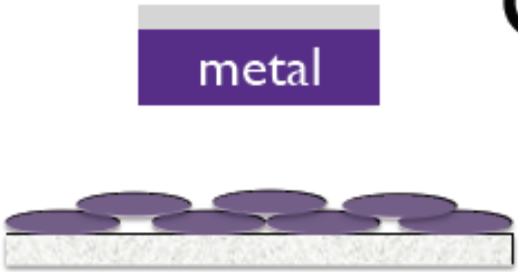
$LaTiO_3/SrTiO_3$

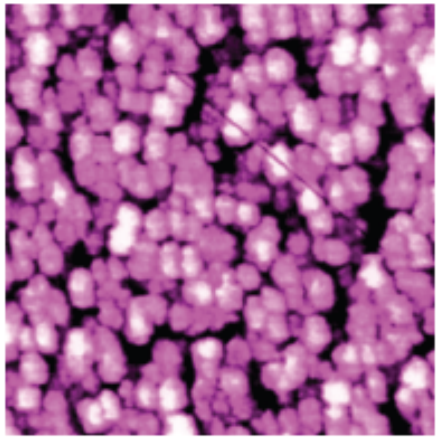
Biscaras et al., PRL 2012

Biscaras et al., Nat. Commun. 2010

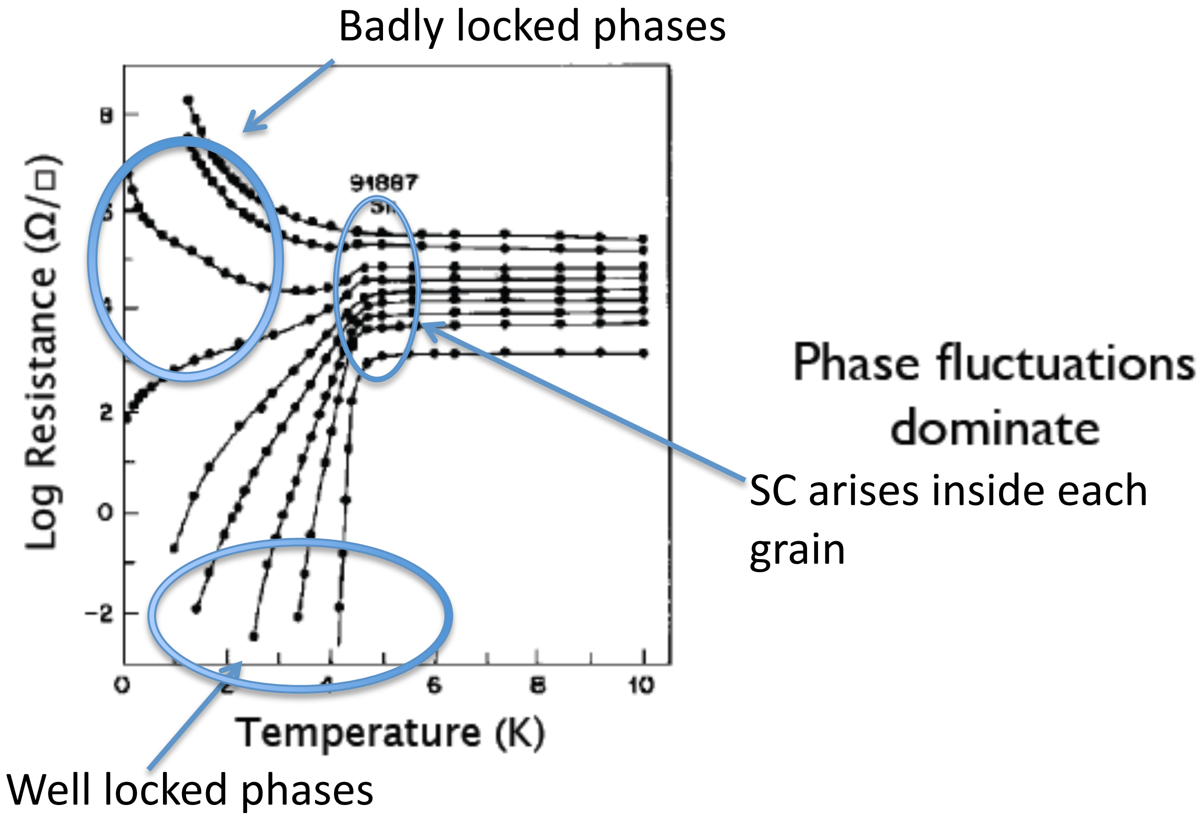
2. Granular SCs: the R-shunted JJA

Granular Film SIT



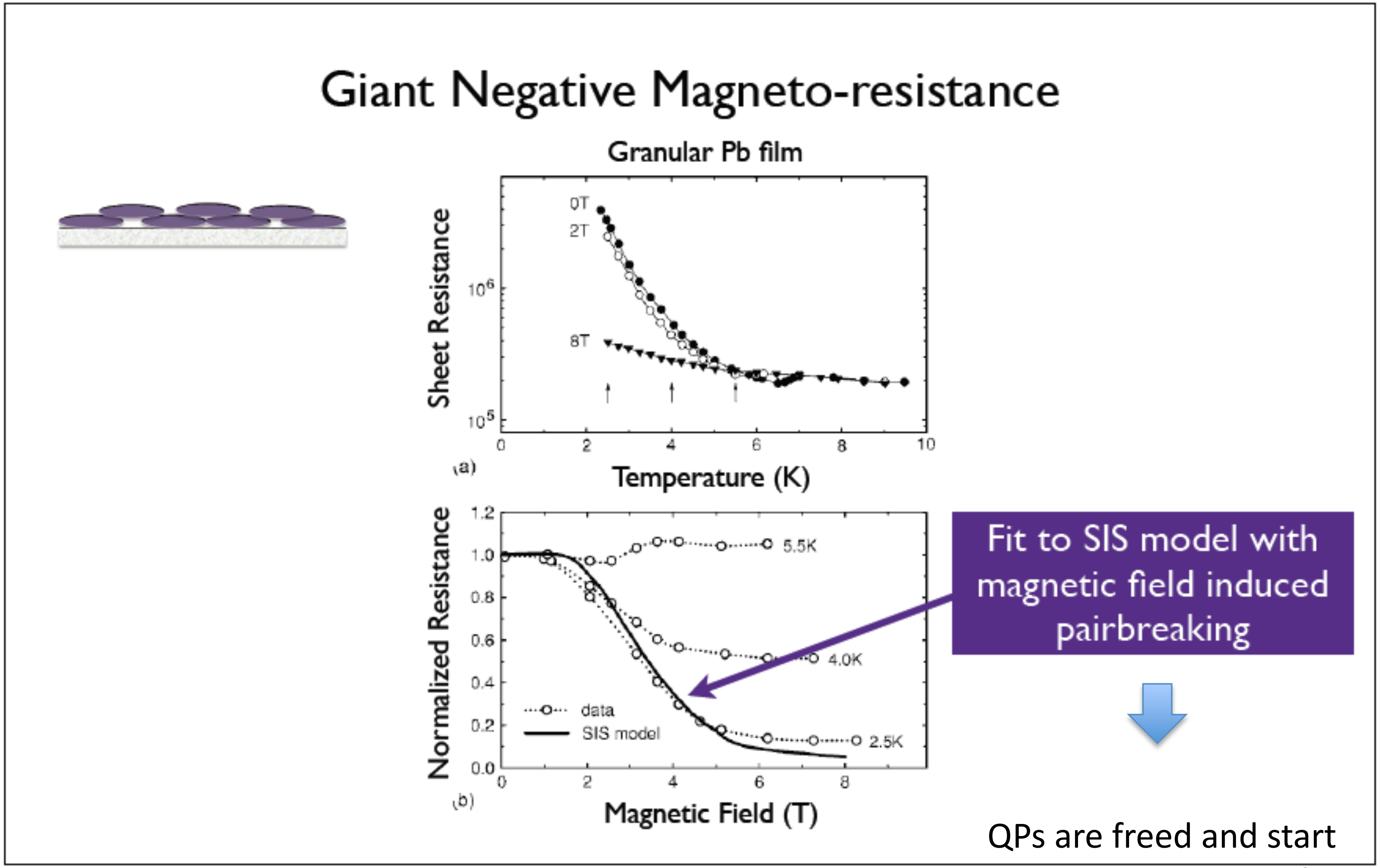


LTSTM image of granular Pb film Ekinci, PRL 1999



Hsu, Valles, Dynes, Garno, Physica B (1994)

J. Valles talk, Leiden



QPs are freed and start conducting jumping from one grain to the other....

3. vs 4. The Fermionic vs. Bosonic scenarios of SIT

Fermionic scenario

Suppression of Superconductivity in homogeneously Disordered Systems

A. M. Finkel'stein, *Physica B* ***197***, 636, (1994)

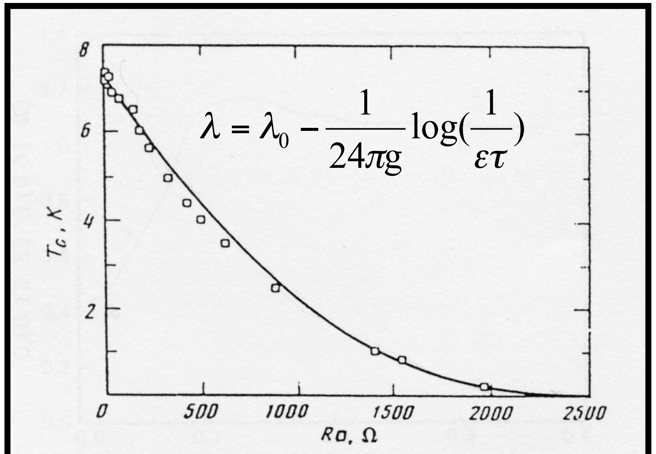


Fig. 14. Suppression of superconductivity in amorphous $Mo_{79}Ge_{21}$ [42]. The solid line is a theoretical fit with Eq. (13).

- Continuous decrease of T_c
- Cooper pairing dies out at the SIT

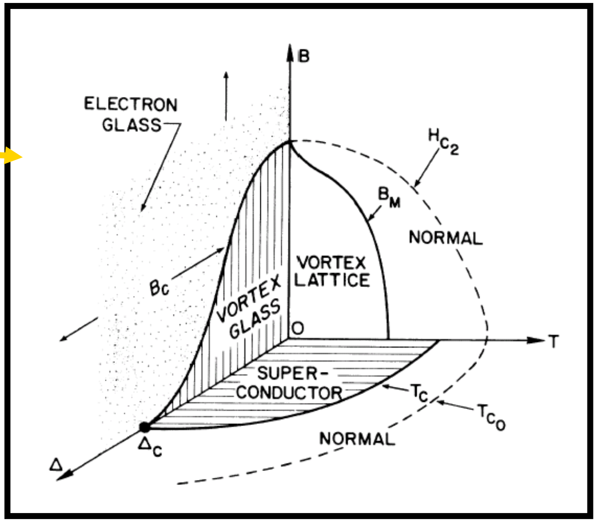
$$\psi = |\psi| e^{i\varphi}$$

Bosonic

Fermionic

Bosonic scenario: old version

M.P.A. Fisher *et al.*, *PRL* **64**, 587 (1990)
M.P.A. Fisher, *PRL* **65**, 923 (1990)



- SIT due to long wave length phase fluctuations
- Insulator made of localized Cooper pairs

Spin models for SC

$$\left[b_k, b_{k'}^+\right] = \delta_{k,k'}\left[1 - \left(n_{k\uparrow} + n_{k\downarrow}\right)\right]$$

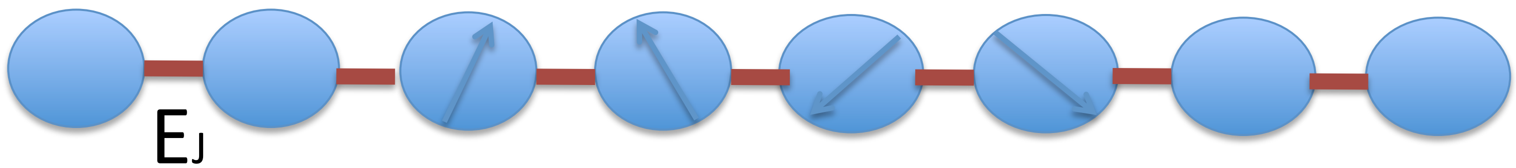
$$\left[b_k, b_{k'}\right] = \left[b_k^+, b_{k'}^+\right] = 0$$

have the same algebra of spin-1/2 operators
P.W. Anderson, Phys. Rev. **110**, 827 (1958); **112**, 1900 (1958)

$$S_k^z = \frac{1}{2}\left[1 - \left(n_{k\uparrow} + n_{k\downarrow}\right)\right]$$

$$S_k^+ \equiv S_k^x + iS_k^y = b_k^+, \qquad S_k^- \equiv S_k^x - iS_k^y = b_k$$

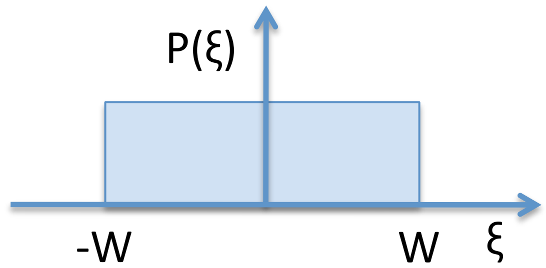
Remember also the XY model representing the 1+1 Josephson-Junction array



Seminal paper: Ma, Lee, PRB 1985

Pair hopping ~ E_J

$$H = -4\sum_i \xi_i S_i^z - g\sum_{i,j}\left(S_i^+ S_j^- + S_i^- S_j^+\right)$$



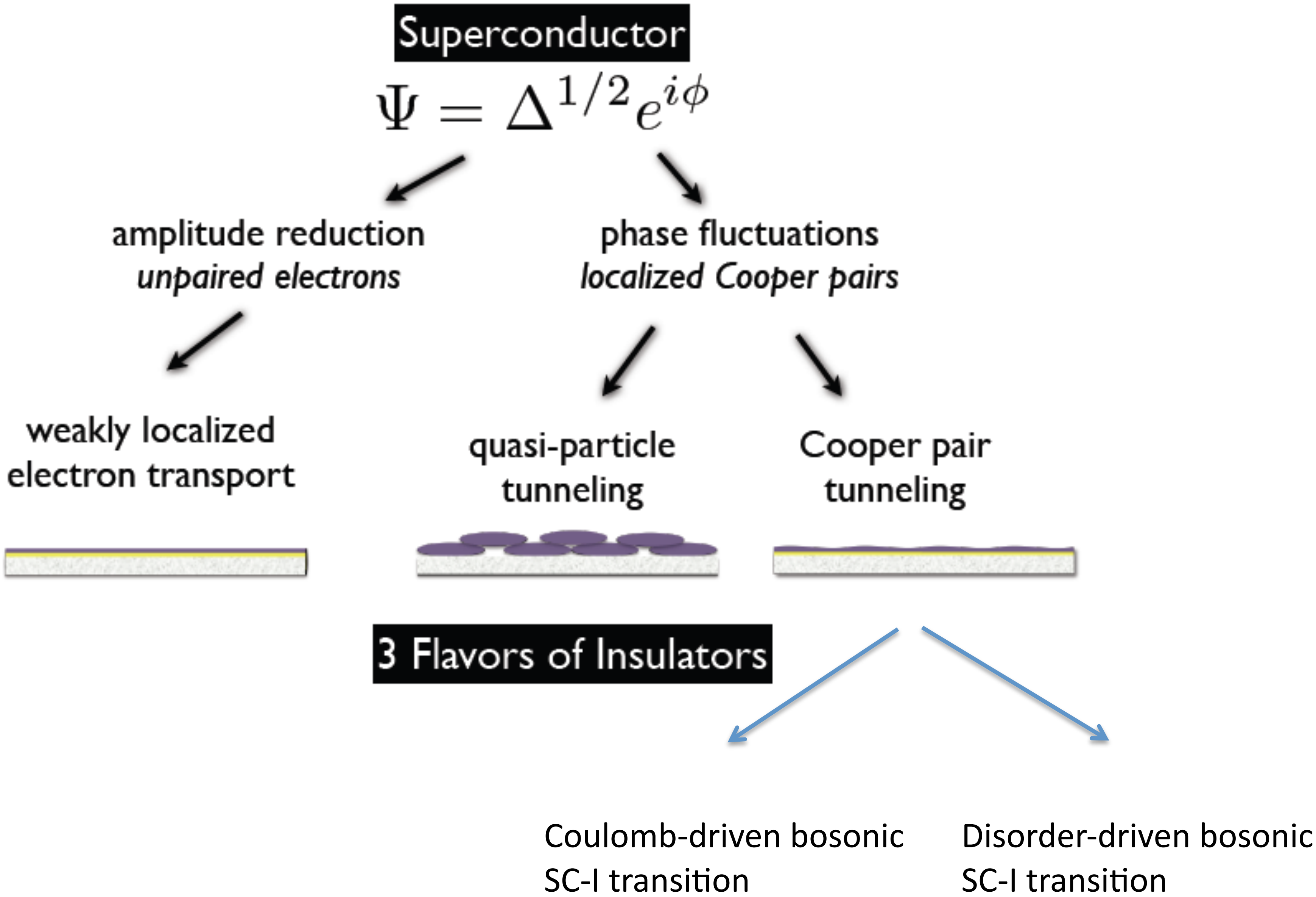
ξ_i are (random) variables: the energy of a spatially localized CP

Two different bosonic scenarios depending on relative importance of disorder (W) and Coulomb rep. (remember $g \sim 1/K \sim \sqrt{E_J/E_C}$ in the XY model for JJA...)

SC means a finite magnetization on the XY plane

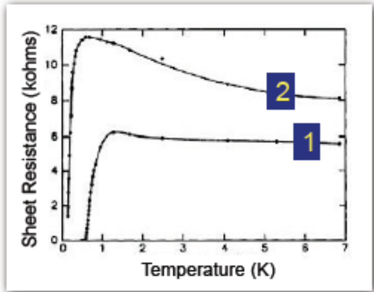
Coulomb-driven SC-I trans (Fisher and Co.)
Disorder not the main issue
long range Coulomb (enhanced by disorder, see Altshuler Aronov) competes with pair hopping $\Rightarrow$ charge rigidity and vortex of xy spin component destroy SC.

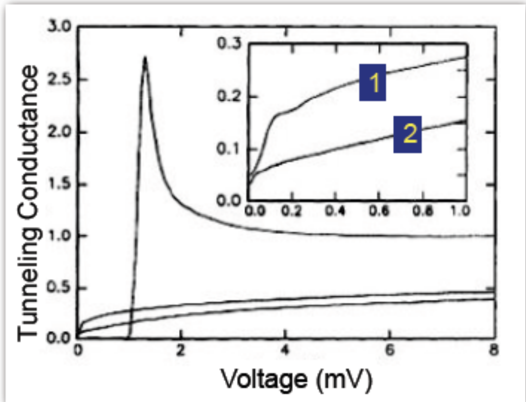
Disorder-driven SC-I transition (Feigel'man, Ioffe, Kravtsov, Mezard,...)
When disorder is too strong (pairs very localized, W>>g) the local field orients the spins along ±z and destroys the "magnetization" (i.e. SC) on the xy plane

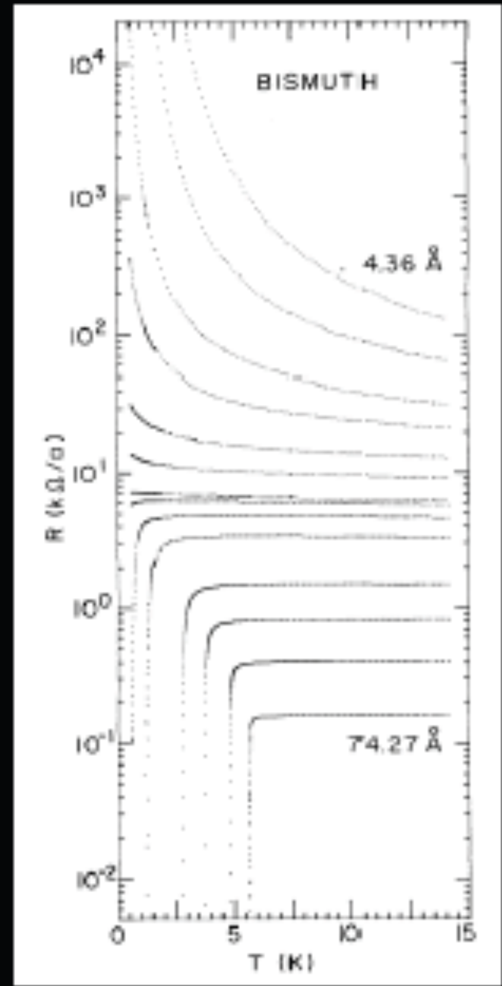

Superconductor
$\Psi = \Delta^{1/2} e^{i\phi}$
amplitude reduction
unpaired electrons
phase fluctuations
localized Cooper pairs
weakly localized
electron transport
quasi-particle
tunneling
Cooper pair
tunneling
3 Flavors of Insulators
Coulomb-driven bosonic
SC-I transition
Disorder-driven bosonic
SC-I transition

An example from the fermionic class:
a-Bi/Ge

Tunneling DOS near SIT







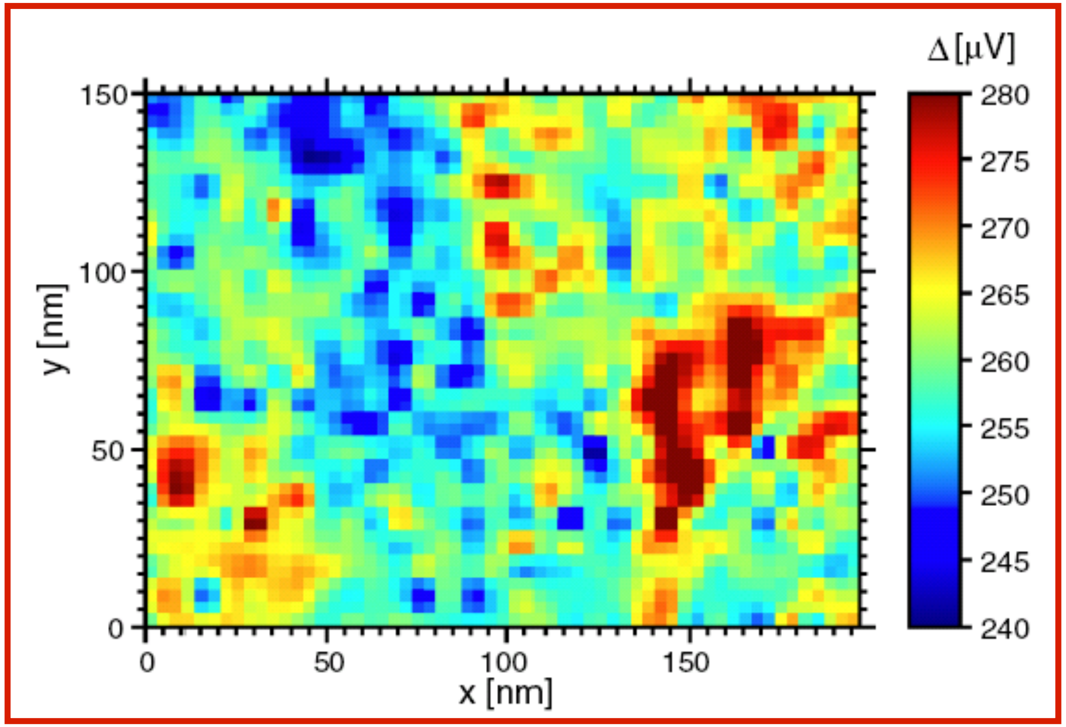
amorphous–Bi films
Haviland, Liu and Goldman

⟹ gap disappears near SIT

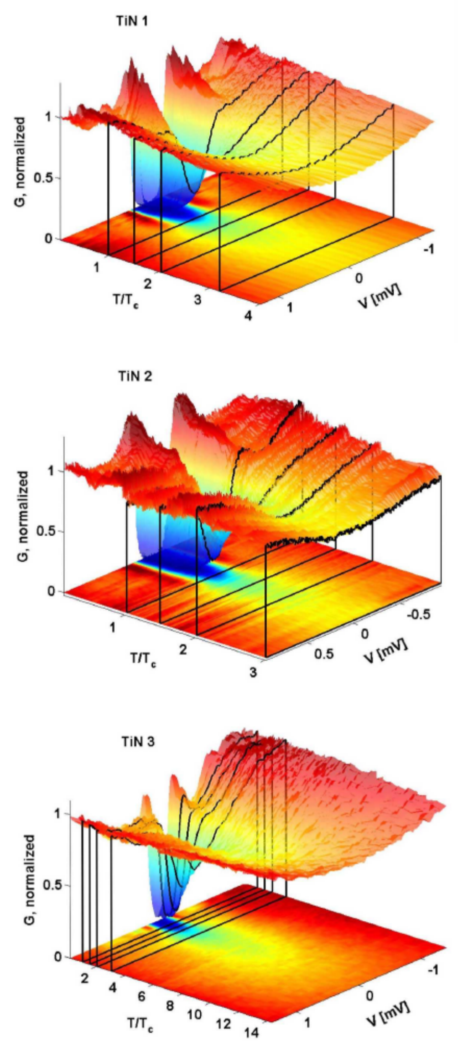
Valles, Dynes, Garno PRL 1992

TiN films

Map of the spectral gap in sample TiN1

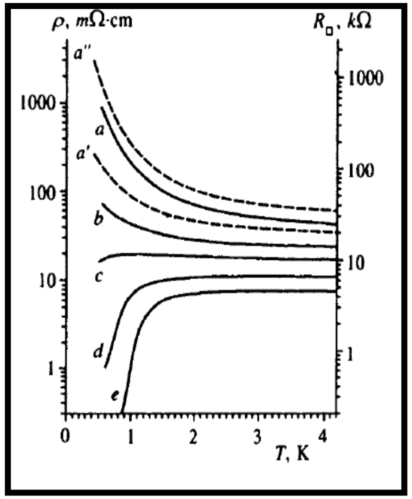


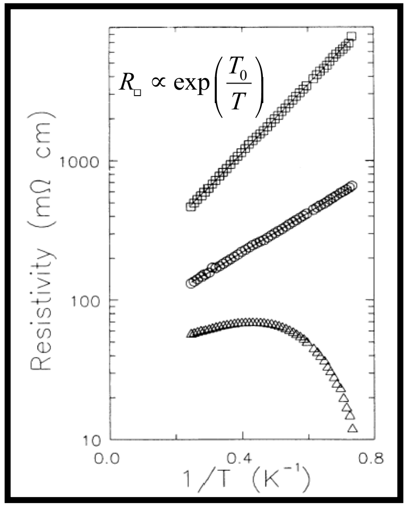
. Sacépé *et al*. PRL **101**, 157006 (2008)

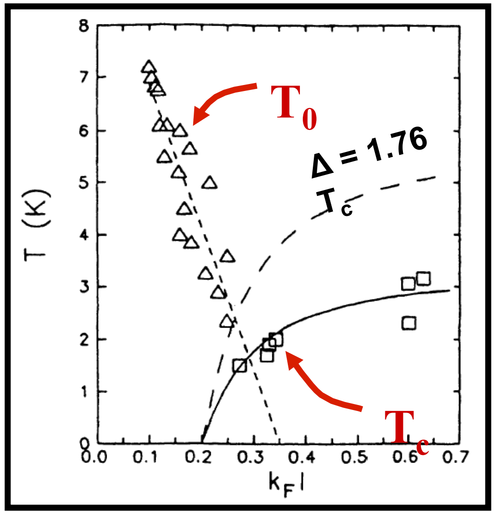


TiN, Sacepe et al. PRL

InOx films

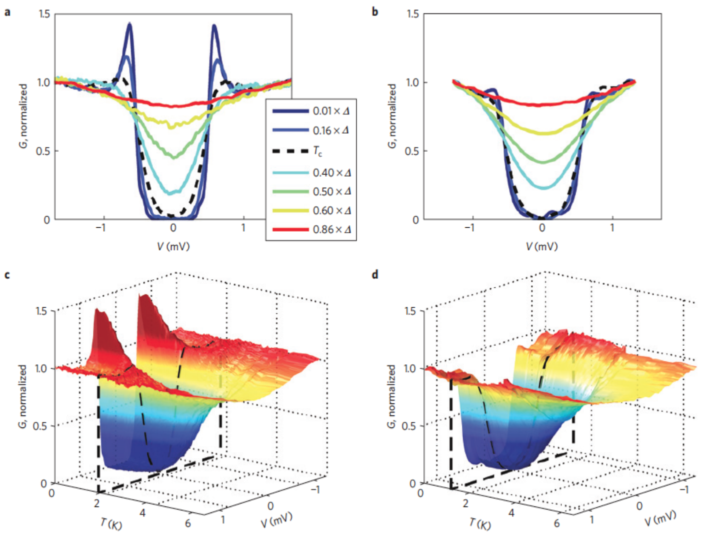






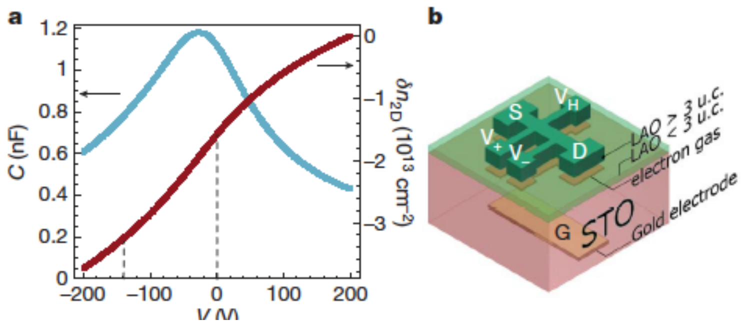
V. F. Gantmakher *et al.*, *JETP* ***82***, 951 (1996)

D. Shahar and Z. Ovadyahu, *Phys. Rev. B* ***46***, 10917 (1992



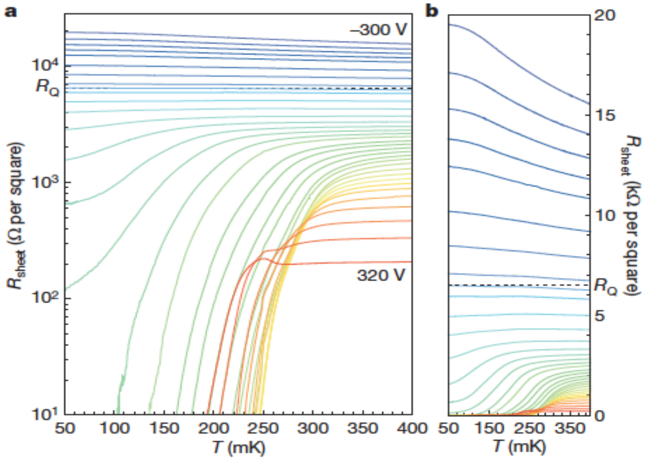
Sacepe et al, Nat. Phys. 2011

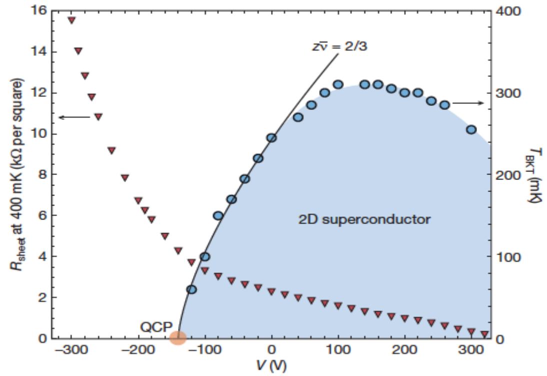
Oxide interfaces



LaAlO$_3$/SrTiO$_3$

Caviglia et al.,Nature 2008

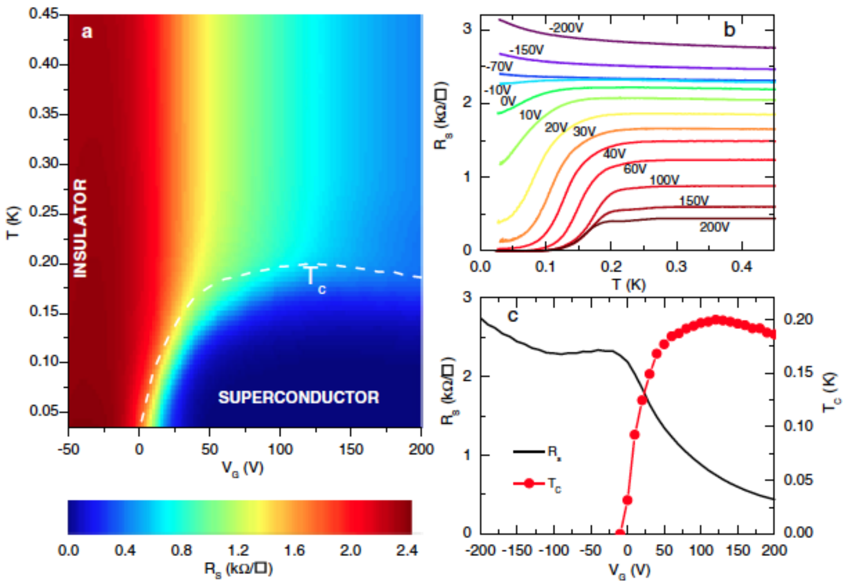


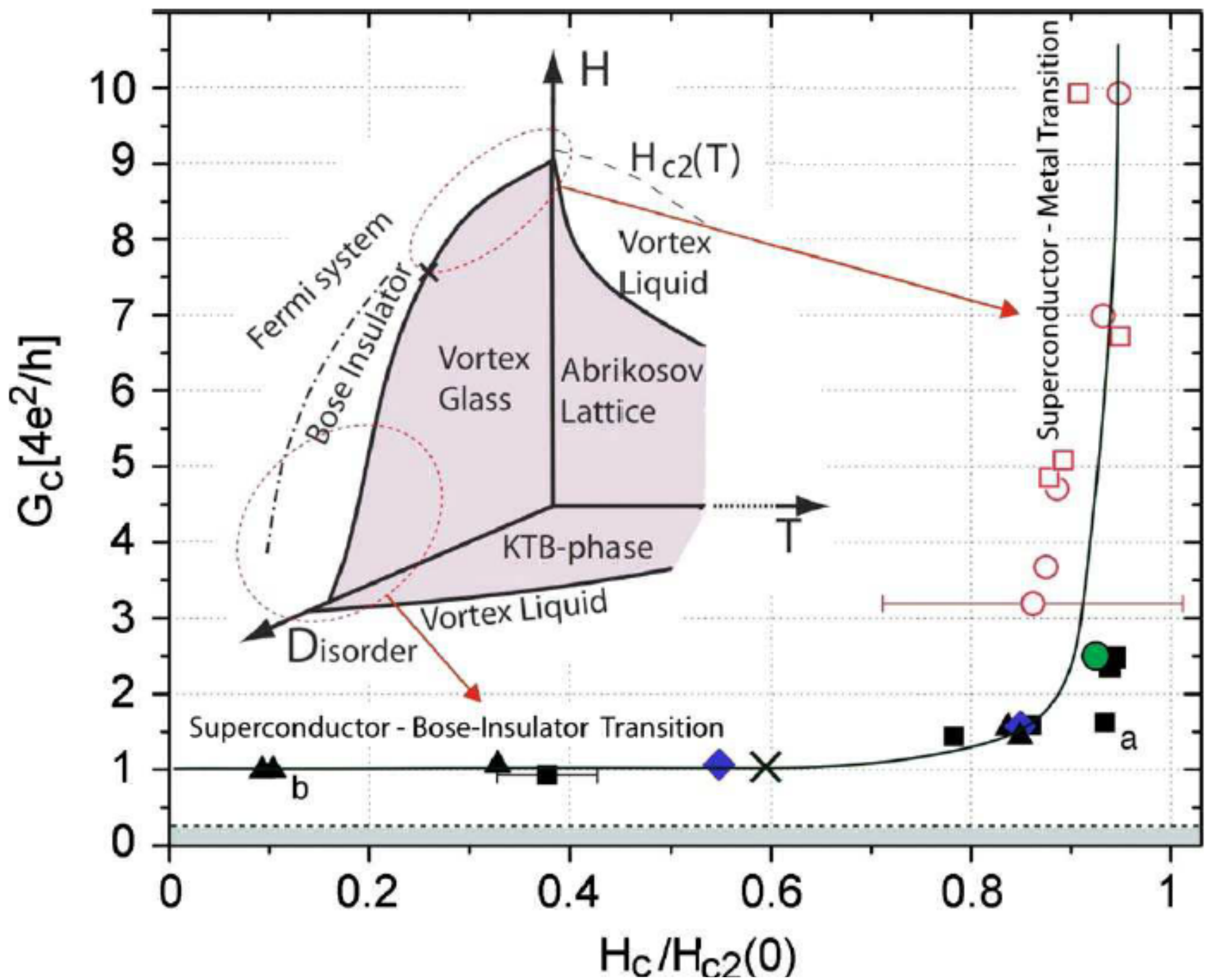


LaTiO$_3$/SrTiO$_3$

Biscaras et al., PRL 2012

Biscaras et al., Nat. Commun. 2010





PHYSICAL REVIEW B 77, 212501 (2008)

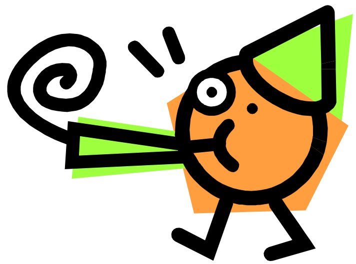
Approach to a superconductor-to-Bose-insulator transition in disordered films

Myles A. Steiner,[1,*] Nicholas P. Breznay,[1] and Aharon Kapitulnik[1,2]

What do we learn:

SC can die in many different ways and all of them can be realized in different systems (or maybe some also in the same system)....

A rich variety of physical systems gives rise to a huge variety of phenomena. Difficult classification and organization of various effects in different materials.



It's still confusing but it's also big fun

Bibliography

•G. Deutscher, *Chance and Matter,* Les Houches, Session XLVI, NATO ASI, 1986, North-Holland

•[Sondhi]S. L. Sondhi, M. Girvin, J. P. Carini, and D. Shahar, Rev. Mod. Phys. **69,** 315 (1997)

•[Chaikin] Chaikin, P.M., and Lubensky T.C., *Principles of Condensed Matter Physics* (Cambridge University, New York), 1995.

•[Goldman] A. M. Goldman, Int. J. Mod. Phys. B **24**, 4081 (2010) Review on SC-I transitions

•http://www.lorentzcenter.nl/lc/web/2011/449/presentations/index.php3?wsid=449&type=presentations