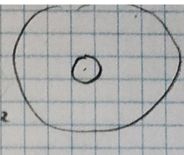
SOLUZIONI

Soluz. Es 1 Rensel

$R = 15 \times 10^{-2}$ m

$\varepsilon_r = 4{,}0$

$s = 0{,}5 \times 10^{-2}$



$Q = 5{,}0 \times 10^{-10}$ C

1)

$$E = \frac{Q}{4\pi\varepsilon_0 a^2 \varepsilon_r} \qquad \frac{1}{4\pi\varepsilon_0} \simeq 9 \times 10^{9}\ \mathrm{Nm^2/C^2}$$

$a = R/2$

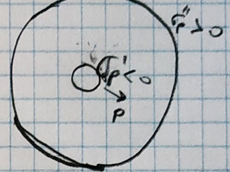
$$E = 200\ \mathrm{V/m}$$

2)

$$E = \frac{Q}{4\pi\varepsilon_0 b^2} \qquad b = 2R \qquad E = 50\ \mathrm{V/m}$$

3)

$$D = \frac{Q}{4\pi r^2} = \varepsilon_0 \varepsilon_r E$$



$$P = \varepsilon_0(\varepsilon_r - 1)E = \frac{\varepsilon_r - 1}{\varepsilon_r} D$$

$$\sigma_p = \vec{P} \cdot \vec{n}_e$$

$$\sigma_p' = -\frac{\varepsilon_r - 1}{\varepsilon_r}\frac{Q}{4\pi s^2} = -1{,}2 \times 10^{-6}\ \mathrm{C/m^2}$$

$$\sigma_p'' = \frac{\varepsilon_r - 1}{\varepsilon_r}\frac{Q}{4\pi R^2} = 1{,}3 \times 10^{-9}\ \mathrm{C/m^2}$$

4)

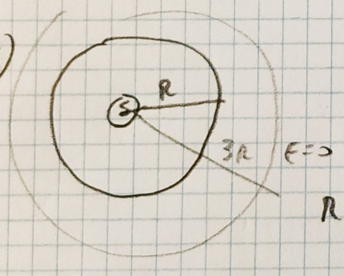
$$\rho_p = -\nabla \cdot \vec{P} = -\frac{\varepsilon_r - 1}{\varepsilon_r}\nabla \cdot D = -\frac{\varepsilon_r - 1}{\varepsilon_r}\rho = 0$$

6)

$$V_R - V_{3R} = V_R = \int_R^{3R} \frac{Q}{4\pi\varepsilon_0 r^2}\,dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{3R}\right) =$$

$$= \frac{Q}{6\pi\varepsilon_0 R} = 20\ \mathrm{V}$$

5)



$r < s \qquad E = 0$

$s < r < R \qquad E = \frac{Q}{4\pi\varepsilon_0\varepsilon_r r^2} \qquad D = \frac{Q}{4\pi r^2}$

$R < r < 3R \qquad E = \frac{Q}{4\pi\varepsilon_0 r^2} \qquad D = \frac{Q}{4\pi r^2}$

$r > 3R \qquad E = 0$

$$U = U_1 + U_2 \qquad U = \int u\,d\tau = 4\pi\int u(r)\,r^2\,dr \qquad u = \frac{E \cdot D}{2}$$

$$U_1 = \int_s^R \frac{4\pi}{2}\frac{Q}{4\pi\varepsilon_0\varepsilon_r}\frac{Q}{4\pi r^2}\,dr = \frac{Q^2}{8\pi\varepsilon_0\varepsilon_r}\left(\frac{1}{s} - \frac{1}{R}\right) = 108{,}6 \simeq 110\ \mathrm{J}$$

$$U_2 = 4\pi\int_R^{3R}\frac{1}{2}\frac{Q}{4\pi\varepsilon_0}\frac{Q}{4\pi r^2}\,dr = \frac{Q^2}{8\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{3R}\right) = 10\ \mathrm{J}$$

$$U_1 + U_2 = 120\ \mathrm{J}$$

CORREZIONE ESERCIZIO 2

$$Ri = \frac{d\Phi(\vec{B})}{dt} = \ell B v \qquad i = \frac{\ell B v}{R}$$ (3)

$$m\ddot{x} = mg - i\ell B \qquad \dot{v} = g - \frac{\ell^2 B^2}{mR} v$$

$$\dot{v} = -A(v - v_\infty) \qquad A = \frac{\ell^2 B^2}{mR} \qquad v_\infty = \frac{g}{A} = \frac{mgR}{\ell^2 B^2}$$ (4)

$$\ln\left(\frac{v(t) - v_\infty}{v(0) - v_\infty}\right) = -At \qquad \overset{v(0)=0}{v(t) = v_\infty\left(1 - e^{-At}\right)}$$ (3)

$$x(t) = \int_0^t dt'\, v_\infty\left(1 - e^{-At'}\right) = v_\infty\left(t + \frac{1}{A}\left(e^{-At} - 1\right)\right) = v_\infty\left(t - \frac{1}{A}\left(1 - e^{-At}\right)\right)$$

(3)

$$v_\infty = \frac{mgR}{\ell^2 B^2} = \frac{10^{-2}\, 9{,}8 \times 10^1}{10^{-2}\, 1} = 98\ m/s$$

(3)

$$i_\infty = \frac{\ell B}{R} v_\infty = \frac{\ell B}{R}\frac{mgR}{\ell^2 B^2} = \frac{mg}{\ell B} = \frac{10^{-2}\, 9{,}8}{10^{-1}\, 1} = 0{,}98\ A$$

16