ESE 1 $a = 10^{-1}\,m$ $\quad b = 10^{-3}\,m$ $\quad Q = 10^{-8}$

$\varepsilon_r = 4 \quad \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$

1 A $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} \quad r > b$

$\vec{E}(\vec{r}) = \vec{0} \quad r < b$

B $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} \quad b < r < a,\ r > 2a$

$\vec{E}(\vec{r}) = \vec{0} \quad r < b,\ a < r < 2a$

C $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} \quad b < r < a,\ r > 2a$

$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{Q}{r^2}\hat{r} \quad a < r < 2a$

$\vec{E}(\vec{r}) = \vec{0} \quad r < b$

D $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} \quad b < r < 2a$

$\vec{E}(\vec{r}) = \vec{0} \quad r < b \quad r > 2a$

[6 PUNTI]

② Ⓐ $V(b) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{b} = \frac{9\times10^9\times10^{-8}}{10^{-3}} = 9\times10^4\ V = 90000\ V$

Ⓑ $V(b) = \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a} + \frac{1}{2a}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{2a}\right) = \frac{90}{10^{-3}}\left(1 - \frac{1}{200}\right)V$

$= 89550\ V$

Ⓒ $V(b) = \frac{1}{4\pi\varepsilon_0}\left[\frac{1}{b} - \left(1 - \frac{1}{\varepsilon_r}\right)\left(\frac{1}{a} - \frac{1}{2a}\right)\right] = \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{\varepsilon_r - 1}{\varepsilon_r}\frac{1}{2a}\right) =$

$= \frac{90}{10^{-3}}\left(1 - \frac{3}{4}\frac{1}{200}\right) = 89662\ V$

Ⓓ $V(b) = \frac{1}{4\pi\varepsilon_0}\left[\frac{1}{b} - \frac{1}{2a}\right] = 89550\ V$

[7 + 2 PUNTI]

③ $ENE = \frac{1}{2}\sum_i^{UB} Q_i V_i$

NEI 4 CASI SULLA SFERA DI RAGGIO b $Q_i = Q$
SUGLI ALTRI CONDUTTORI NEI CASI Ⓐ Ⓑ e Ⓒ
$Q_i = 0$
IN Ⓓ SULLA SFERA DI RAGGIO 2a $Q_i = -Q$
MA SU QUESTA SFERA $V_i = 0$ PERCHÈ È A TERRA
QUINDI IN TUTTI E 4 I CASI

$$ENE = \frac{1}{2} Q \underbrace{V(b)}_{\text{POTENZIALE DELLA SFERA DI RAGGIO b}}$$

Ⓐ :

$$ENE = \frac{1}{8\pi\varepsilon_0}\frac{Q^2}{b} = \frac{9\times10^4\times10^{-8}}{2} = 4{,}5\times10^{-4}\,J$$

Ⓑ $$ENE = \frac{1}{8\pi\varepsilon_0}\frac{Q^2}{b} - \frac{1}{8\pi\varepsilon_0}\frac{Q^2}{2a} = 4{,}4775\times10^{-4}\,J$$

Ⓒ $$ENE = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{b} - \frac{1}{8\pi\varepsilon_0}\frac{\varepsilon_r - 1}{\varepsilon_r}\frac{Q^2}{2a} = 4{,}483\times10^{-4}\,J$$

Ⓓ $$ENE = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{b} - \frac{1}{8\pi\varepsilon_0}\frac{Q^2}{2a} = 4{,}4775\times10^{-4}\,J$$

[6+2 PUNTI]

④ LAVORO = $ENE_{FINALE} - ENE_{INIZIALE}$

STATO FINALE = SFERA DI RAGGIO b ISOLATA DAL RESTO DEL SISTEMA E RESTO DEL SISTEMA SCARICO

⇓

$$ENE_{FINALE} = ENE_{SFERA\ RAGGIO\ b} = \frac{1}{8\pi\varepsilon_0}\frac{Q^2}{b}$$

$ENE_{INIZIALE}$ = ENERGIE CALCOLATE AL PUNTO ③

⇓

Ⓐ LAV = 0

Ⓑ $LAV = \frac{1}{8\pi\varepsilon_0}\frac{Q^2}{2a} = \frac{9 \times 10^9}{4} \frac{\times 10^{-16}}{10^{-1}} = 2{,}25 \times 10^{-6}\ J$

Ⓒ $LAV = \frac{1}{8\pi\varepsilon_0}\frac{\varepsilon_2 - 1}{\varepsilon_2}\frac{Q^2}{2a} = \frac{9}{4}\,\frac{3}{4} \times \frac{10^9 \times 10^{-16}}{10^{-1}} = 1{,}69 \times 10^{-6}\ J$

Ⓓ $LAV = \frac{1}{8\pi\varepsilon_0}\frac{Q^2}{2a} = 2{,}25 \times 10^{-6}\ J$

LAVORO > 0

INDIPENDENTE DAL RAGGIO b

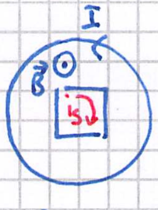
[1 x 2 PUNTI]

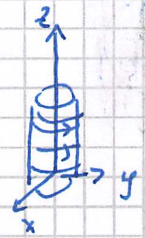
COMPITO 31/1/2020 – ESERCIZIO DI MAGNETISMO

Campo nel solenoide $\vec{B} = \mu_0 n I \hat{z}$ $\quad n = N/\ell$

$\Phi_{spira} = B a^2 = \mu_0 n a^2 I \quad (M_{sol-spir} = \mu_0 n a^2)$

$f_i = -\frac{d\Phi_{spira}}{dt} = -\mu_0 n a^2 \frac{dI}{dt} = -\mu_0 n a^2 I_0 \alpha$





1) $\mu_0 n a^2 I_0 \alpha = r i_s$

$i_s = \frac{\mu_0 n a^2 I_0 \alpha}{r} = 4\pi \cdot 10^{-7} \frac{10^3}{50 \cdot 10^{-2}} \frac{10^{-4}\, 2 \cdot 4}{10^{-3}} = 2 \cdot 10^{-3} A = 2\ mA = i_\infty$

2) $f = \mu_0 n a^2 I_0 \alpha$

$f - L\frac{di_s}{dt} = r i_s$, $\quad i_s(t=0) = 0 \quad i_s(t) = i_\infty (1 - e^{-t/\tau}) \quad \tau = \frac{L}{r} \quad i_\infty = \frac{f}{r}$

Da 1) $\quad i_\infty = 2\ mA$, $\tau = \frac{2 \cdot 10^{-3}}{10^{-3}} = 2\ s \quad$ dopo $t = 4\ s \quad (1 - e^{-t/\tau}) = 0{,}865$

$i_s(t = 4s) = 1{,}7\ mA$

3) $M_{spir-sol} = M_{sol-spir} = \mu_0 n a^2 = M$

4) $i_s = i_0 \cos(\omega t) \quad \Phi_{sol} = M i_s = \mu_0 n a^2 i_0 \sin(\omega t)$

$f_i = -\frac{d\Phi_{sol}}{dt} = -\mu_0 n a^2 i_0 \omega \cos(\omega t)$

$I = \frac{\mu_0 n a^2}{R} i_0 \omega \cos(\omega t) = \frac{i_\infty}{I_0 \alpha} r \frac{i_0 \omega}{R} = i_\infty \frac{r}{R} \frac{i_0}{I_0} \frac{\omega}{\alpha} = i_\infty 10^{-3} \frac{10}{2} \frac{400}{4} = \frac{i_\infty}{2}$

per $\bar{t} = \pi/4\omega \quad \cos(\omega t) = \cos(\pi/4) = 1/\sqrt{2}$

$I(\bar{t}) = \frac{i_\infty}{2\sqrt{2}} = 0{,}7\ mA$