ESERCIZIO CONDENSATORI

CASO DUE DIELETTRICI

1) $0<z<d$ PER GAUSS NEI MEZZI $\Phi(\vec{D}) = Q_{LIB}$

$$\boxed{\vec{D}(\vec{r}) = \hat{z}\,\frac{Q}{S}} \quad (1)$$

2) $\vec{E}(\vec{r}) = \frac{1}{\varepsilon_0\,\varepsilon_r(\vec{r})}\vec{D}(\vec{r}) \Rightarrow$

$$\boxed{\begin{array}{ll} 0<z<\frac{d}{2} & \vec{E}(\vec{r}) = \hat{z}\,\frac{Q}{\varepsilon_0\varepsilon_r^A S} \\ \frac{d}{2}<z<d & \vec{E}(\vec{r}) = \hat{z}\,\frac{Q}{\varepsilon_0\varepsilon_r^B S} \end{array}} \quad (1')$$

3) $\Delta V = V(\vec{0}) - V(\hat{z}d) = -\int_d^0 dz\,E_z = \int_0^d dz\,E_z = \boxed{\frac{dQ}{\varepsilon_0 S}\cdot\frac{1}{2}\left[\frac{1}{\varepsilon_r^A} + \frac{1}{\varepsilon_r^B}\right]}$ (2)

4) PER $z\neq\frac{d}{2}$ $z\neq 0$ $z\neq d$ OVVERO NON SULLE INTERFACCE

$\rho_{POL}(\vec{r}) = \rho(\vec{r}) = \varepsilon_0\,\vec{\nabla}\cdot\vec{E}(\vec{r}) = 0$ $\quad\boxed{\rho_{POL}(\vec{r}) = 0}$

$z=0$ PER GAUSS $\sigma_{LIB} + \sigma_{POL} = \varepsilon_0\,E_z(0^+)$ $\quad\sigma_{LIB} = \frac{Q}{S}$

$$\boxed{\sigma_{POL} = -\sigma_{LIB} + \varepsilon_0 E_z = -\frac{Q}{S} + \frac{1}{\varepsilon_r^B}\frac{Q}{S} = -\left(1-\frac{1}{\varepsilon_r^A}\right)\frac{Q}{S} < 0}$$

$z=\frac{d}{2}$ PER GAUSS $\sigma_{POL} = \varepsilon_0\left(E_z\left(\frac{d}{2}^+\right) - E_z\left(\frac{d}{2}^-\right)\right) = \left(\frac{1}{\varepsilon_r^B} - \frac{1}{\varepsilon_r^A}\right)\frac{Q}{S}$

$z=d$ PER GAUSS $\sigma_{POL} = -\sigma_{LIB} - \varepsilon_0\left(E_z(d^-)\right)$

$$\boxed{\sigma_{POL} = +\left(1-\frac{1}{\varepsilon_r^B}\right)\frac{Q}{S} > 0} \quad (3)$$

5) $C = \frac{Q}{\Delta V}$ $\quad C = \frac{\varepsilon_0 S}{d}\,2\,\frac{\varepsilon_r^A\,\varepsilon_r^B}{\varepsilon_r^A + \varepsilon_r^B}$ (1)

6) $ENE = \frac{1}{2}\frac{1}{C}Q^2 = \frac{d}{\varepsilon_0 S}\,\frac{1}{4}\,\frac{\varepsilon_r^A + \varepsilon_r^B}{\varepsilon_r^A\cdot\varepsilon_r^B}\,Q^2$ (1)

CASO $\varepsilon_r = \varepsilon_r^A + (\varepsilon_r^B - \varepsilon_r^A)\frac{z}{d} = A + Bz$ $\quad A = \varepsilon_r^A$

$B = \frac{\varepsilon_r^B - \varepsilon_r^A}{d}$

1) $\boxed{\vec{D}(\vec{r}) = \hat{z}\frac{Q}{S}}$ (1)

2) $\boxed{\vec{E}(\vec{r}) = \hat{z}\frac{Q}{\varepsilon_0 S}\frac{1}{(Bz+A)}}$ (1)

3) $\Delta V = \int_0^d dz \frac{Q}{\varepsilon_0 S}\frac{1}{(Bz+A)} = \frac{Q}{\varepsilon_0 S B}\ln\left(\frac{Bd+A}{A}\right) = \boxed{\frac{Q\,d}{\varepsilon_0 S(\varepsilon_r^B - \varepsilon_r^A)}\ln\left(\frac{\varepsilon_r^B}{\varepsilon_r^A}\right)}$ (2)

4) $0 < z < d$

$\rho_{pol}(\vec{r}) = \rho(\vec{r}) = \varepsilon_0 \vec{\nabla}\cdot\vec{E}(\vec{r}) = \frac{Q}{S}\frac{d}{dz}\frac{1}{(Bz+A)} = -\frac{Q}{S}\frac{B}{(Bz+A)^2}$

$$\rho_{pol} = -\frac{Q}{Sd}\frac{(\varepsilon_r^B - \varepsilon_r^A)}{\left[\frac{\varepsilon_r^B - \varepsilon_r^A}{d}z + \varepsilon_r^A\right]^2}$$

SULLE INTERFACCE COME CASO PRECEDENTE ($E_1(0^+)$ e $E_2(d^-)$ UGUALI A CASO PRECEDENTE

$z=0$ $\sigma_{pol} = -\left(1 - \frac{1}{\varepsilon_r^A}\right)\frac{Q}{S} < 0$

$z=d$ $\sigma_{pol} = +\left(1 - \frac{1}{\varepsilon_r^B}\right)\frac{Q}{S} > 0$ (3)

5) $C = \frac{Q}{\Delta V}$ $\quad \boxed{C = \frac{\varepsilon_0 S}{d}(\varepsilon_r^A - \varepsilon_r^B)\ln\left(\frac{\varepsilon_r^A}{\varepsilon_r^B}\right)}$ (0)

6) $\boxed{\mathrm{ENE} = \frac{1}{2}CQ^2}$ (0)

Soluzione problema 2

1)

$$L = \mu_0 n^2 l(\pi a^2) = 1.6H$$

2)

$$U = \frac{1}{2} Li(t_0 = 10s)^2 = \frac{1}{2} LIsin(\omega t_0)^2 = 5.4\times10^{-5} J$$

3)

$$2\pi r E_r = \frac{d}{dt}(B\pi r^2) = \pi r^2 \mu_0 n \frac{d}{dt} i(t)$$

$$E_r = \frac{r}{2}\mu_0 n\omega I \cos(\omega t)$$

$$\max(E_r) = \frac{r}{2}\mu_0 n\omega I = 1.3\times10^{-4}\,\mathrm{V/m}$$

4)

$$U_i = \frac{L}{2} I^2$$

$$U_f = \frac{L}{2}(2I)^2$$

$$\Delta U = U_f - U_i = \frac{3}{2} LI^2 = 2.4\times10^{-4} J$$

5)

$$f - L\frac{di}{dt} = Ri$$

$$i(0) = I$$
$$i(t \to \infty) = 2I$$

$$i(t) = I(2 - e^{-t/\tau})$$
$$\tau = L / R$$
$$R = L / \tau = 0.16\Omega$$
$$\mathrm{f=2RI=3.2mV}$$
$$\mathrm{L}=\int_0^{10\tau} fi(t)dt = \int_0^{10\tau} fI(2 - e^{-t/\tau})dt \cong 20 fI\tau = 6.3\mathrm{mJ}$$