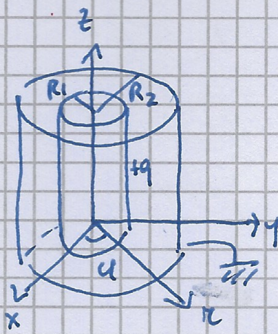
COMPITO 18/12/2020 - ESERCIZIO DI ELETTROSTATICA



Coordinate cilindriche: (r, φ, z) $\quad r = \sqrt{x^2+y^2}$

1) $\vec{E} = \dfrac{q}{2\pi\varepsilon_0 l r}\hat{r} \qquad R_1 < r < R_2$

$\vec{E} = 0 \qquad$ altrimenti

2) $V(R_1) - V(R_2) = V_0 = \displaystyle\int_{R_1}^{R_2} \vec{E}\cdot d\vec{l} = \frac{q}{2\pi\varepsilon_0 l} \ln\frac{R_2}{R_1}$

$q = V_0 \dfrac{2\pi\varepsilon_0 l}{\ln 2} = 4\pi\varepsilon_0 \dfrac{V_0 l}{2\ln 2} = \dfrac{20\cdot 10^{-2}}{2\ln 2}\dfrac{1}{9\cdot 10^{9}} = 1{,}6\cdot 10^{-11}\,C$

$C_0 = \dfrac{q}{V_0} = \dfrac{2\pi\varepsilon_0 l}{\ln 2} = 1{,}6\cdot 10^{-11}\,F = 16\,pF$

3) $\vec{D} = \dfrac{q}{2\pi l r}\hat{r} \qquad \vec{E} = \dfrac{\vec{D}}{\varepsilon_0\varepsilon_r} = \dfrac{q}{2\pi\varepsilon_0\varepsilon_r l r}\hat{r} \qquad \Delta V = \dfrac{V_0}{\varepsilon_r} \Rightarrow C' = \varepsilon_r C_0 = 64\,pF$

4) $\vec{E} = \dfrac{\vec{D}}{\varepsilon_0\varepsilon_r} = \dfrac{q}{2\pi\varepsilon_0 \frac{\alpha}{r} r l}\hat{r} = \dfrac{q}{2\pi\varepsilon_0 \alpha l}\hat{r}$

$V(R_1) - V(R_2) = \Delta V = \dfrac{q}{2\pi\varepsilon_0 \alpha l}(R_2 - R_1)$

$C' = \dfrac{q}{\Delta V} = \dfrac{2\pi\varepsilon_0 l \alpha}{R_2 - R_1} = C_0 \dfrac{\alpha \ln 2}{R_2 - R_1} = C_0 \dfrac{\ln 2 \cdot 2}{1} = 1{,}38\, C_0 = 22\,pF$

5) $U_0 = \dfrac{1}{2}\dfrac{q^2}{C_0} = \dfrac{1}{2}\dfrac{(1{,}6)^2\cdot 10^{-22}}{1{,}6\cdot 10^{-11}} = 0{,}8\cdot 10^{-11}\,J$

$U' = \dfrac{1}{2}\dfrac{q^2}{C'}$

$\Delta U = U_{in} - U_{fin} = U_0 - U' = \dfrac{1}{2}q^2\left(\dfrac{1}{C_0} - \dfrac{1}{C'}\right) = \dfrac{1}{2}\dfrac{q^2}{C_0}\left(1 - \dfrac{C_0}{C'}\right)$

caso 3: $C' = \varepsilon_r C_0 \Rightarrow \Delta U = U_0\left(1 - \dfrac{1}{\varepsilon_r}\right) = U_0 \dfrac{3}{4} = 0{,}6\cdot 10^{-11}\,J$

caso 4: $C' = 1{,}38\, C_0 \Rightarrow \Delta U = U_0\, 0{,}28 = 0{,}22\cdot 10^{-11}\,J$

ESE 2 MAGNETISMO

1) $\vec{B}(\vec{r}) = B(r_\perp)\ \hat{z} \wedge \hat{r}_\perp \qquad \hat{r}_\perp = \frac{\vec{r}_\perp}{|\vec{r}_\perp|} \qquad \vec{r}_\perp = (x, y, 0)$

$0 < r_\perp < a \quad B(r_\perp) = 0$

$$a < r_\perp < 2a \quad B(r_\perp) = \frac{\mu_0}{2\pi} \frac{\cancel{\pi}(r_\perp^2 - a^2)}{r_\perp} \frac{I}{3\cancel{\pi} a^2} =$$

$$= \frac{\mu_0}{6\pi} \left(\frac{r_\perp^2 - a^2}{a^2}\right) \frac{I}{r_\perp} =$$

$$= \frac{\mu_0}{6\pi} \left(\frac{r_\perp}{a^2} - \frac{1}{r_\perp}\right) I$$

$$2a < r_\perp < +\infty \quad B(r_\perp) = \frac{\mu_0}{2\pi} \frac{I}{r_\perp}$$

2) normale alla spira $\perp$ DIREZIONE $\hat{B}$

$$\Phi_{\bigcirc}(\vec{B}) = 0$$

3) USANDO LA CONVENZIONE PER L'ORIENTAMENTO DEL CIRCUITO DATA DALLA FIGURA $\Phi_{\square}(\vec{B}) > 0$

$$\Phi_{\square}(\vec{B}) = l \left[\int_a^{2a} dr_\perp \frac{\mu_0}{6\pi} \left(\frac{r_\perp}{a^2} - \frac{1}{r_\perp}\right) I + \int_{2a}^{10a} dr_\perp \frac{\mu_0}{2\pi} \frac{1}{r_\perp} I \right]$$

$$= \frac{l \mu_0 I}{2\pi} \left[\frac{(4a^2 - a^2)}{3 \times 2 a^2} - \frac{1}{3} \ln 2 + \ln 5 \right]$$

$$= \frac{l \mu_0 I}{2\pi} \left[\frac{1}{2} + \ln 5 - \frac{1}{3} \ln 2 \right]$$

4) $I_O = -\frac{d\Phi_O(\vec{B})}{dt}\frac{1}{R} = 0$

5) $I_{\square} = -\frac{\frac{d\Phi_{\square}(\vec{B})}{dt}}{R} = \frac{l\mu_0}{2\pi}\left[\frac{1}{2} + \ln 5 - \frac{1}{3}\ln 2\right]\frac{\alpha}{R} =$

$= \frac{1 \times 10^{-7} \times 2}{1}\left[1{,}87\right] = 3{,}756 \times 10^{-7}\,A$

$I_{\square} > 0$ CORRENTE PARALLELA