Soluzion esercizio di elettrostatica

- Da $\nabla \cdot \mathbf{E} = \rho\varepsilon_0$,
$$\rho = 2\varepsilon_0 az \tag{1}$$

- Integrando la densità di carica
$$Q = \pi R^2 \int_{-h/2}^{h/2} \rho(z)dz = 0 \tag{2}$$

- Dal teorema di Gauss per il flusso Φ
$$\Phi = \frac{Q}{\varepsilon_0} = 0 \tag{3}$$

- $\mathbf{E} = -\nabla V$ implica che V non dipende da x e y, quindi le basi del cilindro sono a potenziale costante. Integrando si ha
$$V = V(z) = -\frac{a}{3}z^3 - bz + c \tag{4}$$
con c costante arbitraria.

  La differenza di potenziale ΔV tra le due basi è
$$\Delta V = V(h/2) - V(-h/2) = -\frac{a}{12}h^3 - bh = -5200V \tag{5}$$

- $$\rho = \nabla \cdot \mathbf{D} = 2\varepsilon_0\varepsilon_r az \tag{6}$$
$$\rho_p = -\frac{\varepsilon_r - 1}{\varepsilon_r}\rho = -2\varepsilon_0(\varepsilon_r - 1)z \tag{7}$$
$$\sigma_p = \mathbf{P} \cdot \hat{\mathbf{n}} \tag{8}$$
con $\hat{\mathbf{n}}$ la normale uscente dal cilindro, e $\mathbf{P} = \varepsilon_0(\varepsilon_r - 1)\mathbf{E}$.

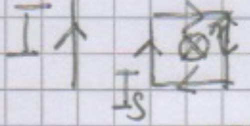
  Si trova quindi $\sigma_p = 0$ sulla superficie laterale del cilindro, mentre sulle basi si ha
$$\sigma_p(\frac{h}{2}) = \sigma_p(-\frac{h}{2}) = \varepsilon_0(\varepsilon_r - 1)(a\frac{h^2}{2} + b) = 2.8 \times 10^{-9}\frac{C}{m^2} \tag{9}$$

$\frac{\mu_0}{4\pi} = 10^{-7}$

$I^0 = 10\,A$
$a = 0.020\,m$

1) $\Phi(\vec{B}) = \frac{\mu_0}{2\pi} I a \int_a^{2a} dr \frac{1}{r} = \frac{\mu_0 I}{2\pi} a \ln 2 = 2\times 10^{-7}\times 2.0\times 10^{-2} \ln 2 \sin(\omega t)$

$= 2{,}77\times 10^{-8} \sin\omega t$



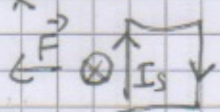
NORMALE PER IL FLUSSO ENTRANTE NEL FOGLIO

2) $A = R$

$I_S R = -\frac{d\Phi}{dt} = -\frac{\mu_0}{2\pi} a \ln 2 \frac{dI}{dt} = -\frac{\mu_0}{2\pi} a \ln 2\, I_0 \omega \cos\omega t$

$I_S = -\frac{\mu_0}{2\pi} a \ln 2 \frac{I_0 \omega \cos\omega t}{R}$

$\vec{F} = \hat{x} F_x$



$d\vec{F} = d\vec{I} \wedge \vec{B}$

$F_x = -\frac{\mu_0 I_0 \sin\omega t}{2\pi}\left(\frac{1}{a} - \frac{1}{2a}\right) a\, I_S$

$F_x = -\frac{\mu_0 I_0 \sin\omega t}{4\pi} I_S \qquad (1)$

$F_x = +\frac{\mu_0^2 I_0^2 a \ln 2\, \omega \sin\omega t \cos\omega t}{R\, 8\pi^2}$

$A = \frac{10^{-14}\times 10^{2}\times 10^{2}\times 2\pi \ln(2)\times 2\times 10^{-2}}{10^{1}}$

$\boxed{A = 1{,}74\times 10^{-12}\,N}$

$= \underbrace{\frac{\mu_0^2 I_0^2 \omega a \ln 2}{R\, 16\pi^2}}_{=A} \sin 2\omega t$

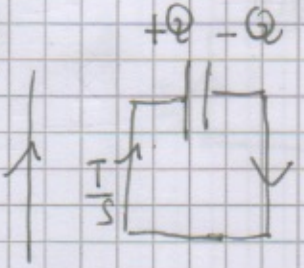
10^{-2} ↓ $2\pi \frac{\omega}{2\pi} t =$ → 2π ($t = 10\,ms$); $t = 1.25\,10^{-3}\,s$ → $\frac{2\pi}{8} = \frac{\pi}{4}$

$\langle F_x \rangle = 0$ PERCHÈ $\frac{1}{T}\int_0^T \sin 2\omega t\, dt = 0$

$\sin 2\omega(10\,ms) = \sin 4\pi = 0 \quad F_x = 0$

$\sin 2\omega(1.25\,ms) = \sin\frac{\pi}{2} = 1 \quad F_x = +1{,}74\times 10^{-12}\,N$

3)



$$Q = C\Delta V$$

$$Q = C\left(-\frac{d\Phi}{dt}\right) = -\frac{\mu_0}{2\pi} a \ln 2\, I_0\, C\omega \cos\omega t$$

$$I_S = \frac{dQ}{dt} = +\frac{\mu_0}{2\pi} a \ln 2\, I_0\, C\omega^2 \sin\omega t$$

DALLA (1)

$$F_x = -\frac{\mu_0 I_0 \sin\omega t}{4\pi a} \frac{\mu_0}{2\pi} a \ln 2\, I_0\, C\omega^2 \sin\omega t$$

$$F_x = -\mu_0^2 I_0^2 C\omega^2 a \frac{\ln 2}{8\pi^2} (\sin\omega t)^2$$

$B = 10^{-14} \times 10^{2} \times 10^{-6} \times 10^{4} \times 2 \times (2\pi)^2 \times 2 \times 10^{-2} \times \ln 2 = 1{,}09 \times 10^{-12}\ N$

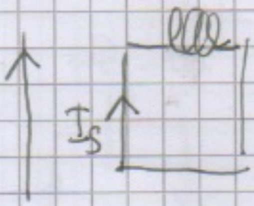
$$\frac{1}{T}\int_0^T dt \sin^2\omega t = \frac{1}{2}$$

$$\langle F_x \rangle = -\mu_0^2 I_0^2 C\omega^2 a \frac{\ln 2}{8\pi^2} \frac{1}{2} = -0.55 \times 10^{-12}\ N$$

$\sin\omega(10\,ms) = 0$

$\sin\omega(1.25\,ms) = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $\qquad F_x = -0.55 \times 10^{-12}\ N$

4)



$$+L\frac{dI_S}{dt} = -\frac{d\Phi}{dt} = -\frac{\mu_0}{2\pi} a \ln 2 \frac{dI}{dt}$$

$$L I_S = -\frac{\mu_0}{2\pi} a \ln 2 \, I$$

$$I_S = -\frac{\mu_0}{2\pi} a \ln 2 \frac{I_0}{L} \sin\omega t$$

DA EQ (1)

$$F_x = \frac{\mu_0 I_0 \sin\omega t}{4\pi} \frac{\mu_0 a}{2\pi} \ln 2 \frac{I_0}{L} \sin\omega t$$

$$F_x = \frac{\mu_0^2 I_0^2}{L} a \frac{\ln 2}{8\pi^2} (\sin\omega t)^2$$

$$C = \frac{10^{-14} \times 10^{2} \times 2 \times 2}{10^{-5}} \times 10^{-2} \ln 2 = $$

$$= 2{,}77 \times 10^{-9} N$$

$$\langle F_x \rangle = \frac{C}{2} = +1{,}39 \times 10^{-9} N$$

$F_x(10 ms) = 0$ $\quad$ $F_x(1{,}25 ms) = \frac{C}{2} = +1{,}39 \times 10^{-9} N$