20 punti (senza domanda opzionale)

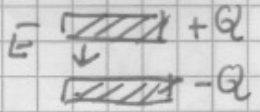
ESE 1 CONDENSATORE ELETTROLITICO

- $C = \frac{S \varepsilon_0 \varepsilon_r}{d}$ $\qquad \sigma = \frac{CV}{S}$ $\qquad E_{ELE} = \frac{CV^2}{2} = \frac{S\varepsilon_0\varepsilon_r}{2d} V^2$

$\sigma = \frac{\varepsilon_0 \varepsilon_r}{d} V$

- $C = \frac{10 \times 8{,}85 \times 10^{-12} \times 10}{10^{-9}} = 0{,}885 \text{ F}$

$\sigma = 0{,}885 \frac{C}{m^2}$ $\qquad E_{ELE} = \frac{88{,}5}{2} = 44{,}3 \text{ J}$



- $-\frac{d}{2} < y < \frac{d}{2}$ NON CI SONO CARICHE $\Rightarrow$ CAMPO COSTANTE

$E(y) = -E_0$

$\frac{d}{2} < y < \frac{d}{2} + a$ CARICA UNIFORME $\rho = \frac{Q}{Sa}$

$\vec{\nabla} \cdot \vec{E} = \frac{\rho(\vec{r})}{\varepsilon_0}$

$\frac{dE(y)}{dy} = \frac{1}{\varepsilon_0} \frac{Q}{Sa}$ $\qquad E(y) = \frac{Q}{\varepsilon_0 Sa}\left(y - \frac{d}{2}\right) + C$

PER $y = \frac{d}{2}$ $D(y)$ CONTINUO $\Rightarrow$ $\varepsilon_0 \varepsilon_r (-E_0) = \varepsilon_0 (\ 0 \ + C)$

$C = -E_0 \varepsilon_r$ $\qquad E(y) = \frac{Q}{\varepsilon_0 Sa}\left(y - \frac{d}{2}\right) - E_0 \varepsilon_r$

$y > \frac{d}{2} + a$. NON CI SONO CARICHE $\Rightarrow$ CAMPO COSTANTE

INTERNO CONDUTTORE $\Rightarrow$ $E(y) = 0$

$y = \frac{d}{2} + a$
$D(y) = \varepsilon_0 E(y)$
CONTINUO $\Rightarrow$ $\frac{Q}{\varepsilon_0 Sa}(a) - E_0 \varepsilon_r = 0$ $\qquad E_0 = \frac{Q}{\varepsilon_0 \varepsilon_r S}$

ANALOGAMENTE PER $y<0$: $\boxed{E(-y)=E(y)}$

RIASSUMENDO

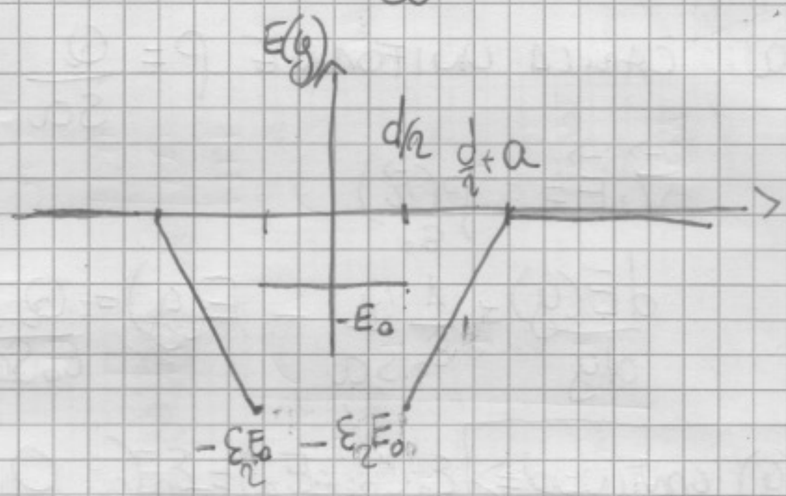
$$E_0 = \frac{Q}{\varepsilon_r \varepsilon_0 S}$$

$y < -\left(\frac{d}{2}+a\right)$ $\quad E(y)=0$

$-\left(\frac{d}{2}+a\right) < y < -\frac{d}{2}$ $\quad E(y) = \varepsilon_r \frac{E_0}{a}\left(-y+\frac{d}{2}\right) - \varepsilon_r E_0$

$-\frac{d}{2} < y < \frac{d}{2}$ $\quad E(y) = -E_0$

$\frac{d}{2} < y < \frac{d}{2}+a$ $\quad E(y) = \varepsilon_r \frac{E_0}{a}\left(y-\frac{d}{2}\right) - \varepsilon_r E_0$



$$V(y) = -\int_0^y dy' E(y') \qquad \text{PER LA [illegible]ZIONE } V(0)=0$$

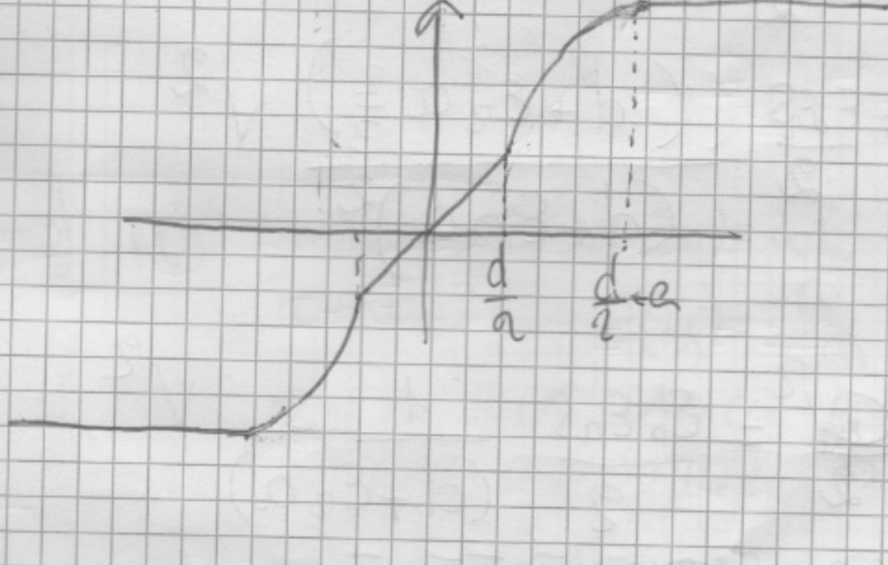
$$-\frac{d}{2} < y < \frac{d}{2} \qquad V(y) = E_0 y$$

$$\frac{d}{2} < y < \frac{d}{2}+a \qquad V(y) = E_0\frac{d}{2} - \frac{\varepsilon_r E_0}{a}\left[\frac{y^2}{2} - \frac{d^2}{8}\right] + \frac{\varepsilon_r E_0}{a}\left(\frac{d}{2}+a\right)\left[y - \frac{d}{2}\right]$$

$$y > \frac{d}{2}+a$$

$$V(y) = V\left(\frac{d}{2}+a\right) = E_0\frac{d}{2} + \frac{\varepsilon_r E_0}{a}\frac{1}{2}\left[\left(\frac{d}{2}+a\right)^2 + \frac{d^2}{4} - \frac{d^2}{2} - da\right]$$

$$V(y) = E_0\frac{d}{2} + \varepsilon_r E_0 \frac{a}{2}$$



$$V = E_0 (d + \varepsilon_r a)$$

$$C_{ELET} = \frac{Q}{V} = \frac{Q}{E_0 (d + \varepsilon_r a)} = \frac{\varepsilon_0 \varepsilon_r S}{d + \varepsilon_r a}$$

FRA LE PLACCHE

$$E^{A}_{ELETR} = \frac{Sd}{2} E_0^2 \varepsilon_0 \varepsilon_r$$

NELLE REGIONI $\frac{d}{2} < y < \frac{d}{2} + a \quad -\left(\frac{d}{2} + a\right) < y < -\frac{d}{2}$

DUE REGIONI

$$E^{B}_{ELETR} = 2 \times \frac{\varepsilon_0}{2} S \int_0^a dt \left(\frac{E_0 \varepsilon_r}{a}\right)^2 t^2$$

$$= \varepsilon_0 E_0^2 \varepsilon_r^2 \frac{a}{3} S$$

$$E_0^2 = \frac{Q^2}{\varepsilon_r^2 \varepsilon_0^2 S^2}$$

$$E_{ELETR} = \frac{\varepsilon_0 S E_0^2}{2} \left(\varepsilon_r d + \varepsilon_r^2 a \frac{2}{3}\right)$$

$$= \frac{\varepsilon_0 \varepsilon_r S}{2} \frac{\left(d + \varepsilon_r a \frac{2}{3}\right)}{(d + \varepsilon_r a)^2} V^2$$

$$E_{GEN} = \frac{C_{EL} V^2}{2} = \frac{\varepsilon_0 \varepsilon_r S}{2} \frac{1}{(d + \varepsilon_r a)} V^2$$

SI MISURA FACILMENTE E_{GEN}

LE DUE ENERGIE SONO DIVERSE PERCHÈ OLTRE ALL'ENERGIA ELETTROSTATICA BISOGNA CONSIDERARE L'ENERGIA CHE CAUSA LA DELOCALIZZAZIONE DELLE CARICHE SULLO SPESSORE DI PELLE

$$C_{ELETTR} = \frac{1}{3} C = \frac{0.885}{3} F =$$

$$E_{GEN} = \frac{44{,}3}{3} J =$$

$$E_{ELETTR} = E_{GEN} \frac{\left(d + \varepsilon_2 a \frac{2}{3}\right)}{(d + \varepsilon_2 a)} = E_{GEN}\left(\frac{1 + \frac{4}{3}}{1+2}\right)$$

$$= E_{GEN}\left(\frac{7}{9}\right) = \frac{44{,}3 \times 7}{27} J =$$

Soluzione esercizio di magnetismo (punti 15)

- Il flusso é nullo
- $$\mathbf{D} = \varepsilon_0 \varepsilon_r \mathbf{E} + \varepsilon_0(\varepsilon_r - 1)\omega B \mathbf{r}_\perp \tag{1}$$
- Da $\nabla \cdot \mathbf{D} = 0$
$$\rho_P = -2\varepsilon_0 \frac{\varepsilon_r - 1}{\varepsilon_r} \omega B \simeq -6.5 \times 10^{-11}\, C\, m^{-3} \tag{2}$$
- La carica di polarizzazione superficiale è presente solo sulla superficie laterale. Dal teorema di Gauss
$$\sigma_P = \varepsilon_0 \frac{\varepsilon_r - 1}{\varepsilon_r} \omega B R \simeq 6.5 \times 10^{-13} C\, m^{-2} \tag{3}$$
- Carica totale di volume
$$Q_V = \pi R^2 L \rho_p \simeq -2.4 \times 10^{-14} C \tag{4}$$
- Carica totale di superficie
$$Q_S = 2\pi R L \sigma_p = -Q_P \simeq 2.4 \times 10^{-14} C \tag{5}$$