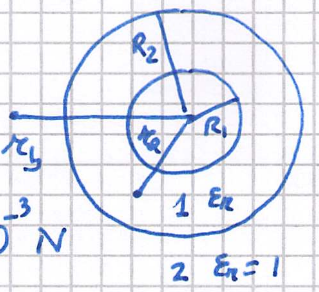
SOLUZIONI ESONERO 27-11-2019

1) $\vec{D}(\vec{r}) = \frac{Q}{4\pi r^2}\hat{r} \qquad r \geq R_1$



$\vec{E}_1(\vec{r}) = \frac{Q}{4\pi\varepsilon_0\varepsilon_r r^2}\hat{r} \qquad \vec{E}_2(\vec{r}) = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$

$F_a = qE_1(r_a) = \frac{qQ}{4\pi\varepsilon_0\varepsilon_r r_a^2} = \frac{9\cdot10^9\ 4\cdot10^{-15}}{4\ 36\cdot10^{-4}} = 2{,}5\cdot10^{-3}\,N$

$F_b = qE_2(r_b) = \frac{qQ}{4\pi\varepsilon_0 r_b^2} = F_a\cdot4\left(\frac{r_a}{r_b}\right)^2 = 1{,}44\,F_a = 3{,}6\cdot10^{-3}\,N$

2) $U_1 = \frac{1}{2}\int_{V_1} d\tau\ \vec{E}_1\cdot\vec{D} = \frac{1}{2}\left(\frac{Q}{4\pi}\right)^2\frac{1}{\varepsilon_0\varepsilon_r}\int_{R_1}^{R_2} 4\pi r^2 dr\ \frac{1}{r^4} = \frac{Q^2}{8\pi\varepsilon_0\varepsilon_r}\left(-\frac{1}{R_2}+\frac{1}{R_1}\right)$

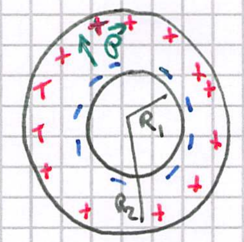
$= 9\cdot10^9\ \frac{16\cdot10^{-12}}{2\cdot4}\left(\frac{1}{4}-\frac{1}{8}\right)\frac{1}{10^{-2}} = \frac{18}{8}10^{-1}\,J = 0{,}225\,J$

$U_2 = \frac{1}{2}\int_{V_2} d\tau\ \vec{E}_2\cdot\vec{D} = \frac{Q^2}{8\pi\varepsilon_0}\frac{1}{R_2} = 9\cdot10^9\ \frac{16\cdot10^{-12}}{2\cdot8\cdot10^{-2}} = 0{,}9\,J$

$U = U_1 + U_2 = 1{,}125\,J$

3) $\vec{P} = \varepsilon_0(\varepsilon_r-1)\vec{E}_1 = \frac{(\varepsilon_r-1)}{\varepsilon_r}\vec{D} \qquad$ in V_1 $\quad \vec{\nabla}\cdot\vec{D}=0 \Rightarrow \rho_p = -\vec{\nabla}\cdot\vec{P} = 0$

$\sigma_p(r=R_1) = \vec{P}\cdot\hat{n}_1 = \vec{P}\cdot(-\hat{r}) = -P =$



$= -\frac{(\varepsilon_r-1)}{\varepsilon_r}\frac{Q}{4\pi R_1^2} = -\frac{3}{4}\frac{4\cdot10^{-6}}{4\pi\,16\cdot10^{-4}}\frac{C}{m^2} =$

$= -1{,}5\cdot10^{-4}\,C/m^2$

$\sigma_p(r=R_2) = \vec{P}\cdot\hat{n}_2 = \vec{P}\cdot\hat{r} = +P = \frac{\varepsilon_r-1}{\varepsilon_r}\frac{Q}{4\pi R_2^2} =$

$= 1{,}5\cdot10^{-4}\left(\frac{R_1}{R_2}\right)^2 C/m^2 = 0{,}375\cdot10^{-4}\,C/m^2$

4) Il campo per $r > R_2$ è nullo, pertanto $F_b = 0$ e si perde il contributo U_2 all'energia elettrostatica

F_a invariato

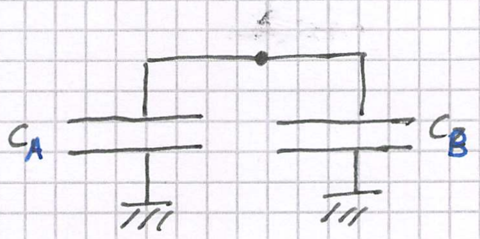
$F_b = 0$

$U = U_1$

5) $V^A(R_1) = V^B(R_1)$ -> unico conduttore, stesso potenziale

$V^A(R_2) = V^B(R_2) = 0$ -> entrambi i gusci esterni sono messi a terra

6) Il sistema è equivalente a due condensatori in parallelo, dei quali inizialmente solo C_A è carico.



Nel sistema A isolato

$$V(R_1) - V(R_2) = \int_{R_1}^{R_2} \vec{E}_1 \cdot d\vec{l} = \frac{Q}{4\pi\varepsilon_0\varepsilon_r}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{Q}{C_A} = \Delta V$$

$$C_A = 4\pi\varepsilon_0\varepsilon_r \frac{R_1 R_2}{R_2 - R_1}$$

$$C_B = \frac{C_A}{\varepsilon_r}$$

Dopo il collegamento

$$\begin{cases} \dfrac{Q_A}{C_A} = \dfrac{Q_B}{C_B} \\ Q_A + Q_B = Q \end{cases} \qquad Q_A = \frac{C_A}{C_B} Q_B = \varepsilon_r Q_B \qquad (\varepsilon_r + 1) Q_B = Q \qquad Q_B = \frac{Q}{5} = 0{,}8 \; 10^{-6}\,C \qquad Q_A = 3{,}2 \; 10^{-6}\,C$$

7) $$\Delta V = \frac{Q_A}{C_A} = \frac{Q_A}{4\pi\varepsilon_0\varepsilon_r} \frac{R_2 - R_1}{R_1 R_2} = 9 \cdot 10^9 \, \frac{3{,}2 \cdot 10^{-6}}{4} \, \frac{4}{32 \cdot 10^{-2}} = 9 \cdot 10^4 \, V$$