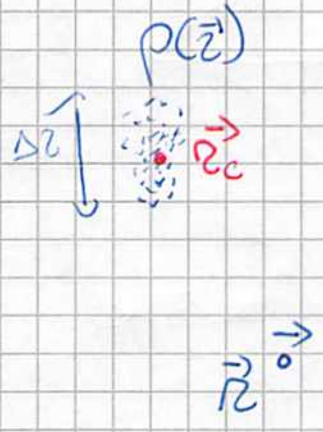
MOMENTI DI UNA DISTRIBUZIONE DI CARICA



ΔR ESTENSIONE DELLA DISTRIBUZIONE

- VOGLIAMO CALCOLARE $V(\vec{r})$ E $\vec{E}(\vec{r})$ IN $\vec{r}$ LONTANO DA $\vec{r}_c$ OVVERO QUANDO

$$|(\vec{r} - \vec{r}_c)| \gg \Delta R$$

IDENTITÀ MATEMATICHE

- $$\frac{1}{(1+\delta)^{1/2}} = 1 - \frac{1}{2}\delta + O(\delta^2)$$

- $|\vec{A}| \gg |\vec{B}|$: $$\frac{1}{|\vec{A}-\vec{B}|} = \frac{1}{(A^2 - 2\vec{A}\cdot\vec{B} + B^2)^{1/2}} = \frac{1}{A}\frac{1}{\Big[\underbrace{1 - \frac{2\vec{A}\cdot\vec{B}}{A^2} + \frac{B^2}{A^2}}_{\delta}\Big]^{1/2}}$$

$$= \frac{1}{A}\left[1 + \frac{\vec{A}\cdot\vec{B}}{A^2} + O\left(\frac{B^2}{A^2}\right)\right] = \frac{1}{A} + \frac{\vec{A}\cdot\vec{B}}{A^3} + O\left(\frac{B^2}{A^3}\right)$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int d^3r' \frac{\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} = \frac{1}{4\pi\varepsilon_0}\int d^3r' \frac{\rho(\vec{r}\,')}{|\underbrace{(\vec{r}-\vec{r}_c)}_{\vec{A}} - \underbrace{(\vec{r}\,'-\vec{r}_c)}_{\vec{B}}|} =$$

$\left(|\vec{B}| \sim r_c \quad A \gg r_c\right)$

PRODOTTO SCALARE

$$\simeq \frac{1}{4\pi\varepsilon_0}\frac{\int d^3r' \rho(\vec{r}\,')}{|\vec{r}-\vec{r}_c|} + \frac{1}{4\pi\varepsilon_0}\frac{(\vec{r}-\vec{r}_c)}{|r-r_c|^3}\cdot\int d^3r' (\vec{r}\,'-\vec{r}_c)\rho(\vec{r}\,')$$

$Q = \int d^3r\, \rho(\vec{r}) =$ MONOPOLO ELETTRICO

$\vec{d} = \int d^3r\, (\vec{r}-\vec{r}_c)\rho(\vec{r}) =$ DIPOLO ELETTICO

$$V(\vec{r}) \simeq \frac{1}{4\pi\varepsilon_0}\frac{Q}{|\vec{r}-\vec{r}_c|} + \frac{1}{4\pi\varepsilon_0}\frac{\vec{d}\cdot(\vec{r}-\vec{r}_c)}{|\vec{r}-\vec{r}_c|}$$

- NOTARE CHE SE $Q \neq 0$ MONOPOLO DOMINA E DIPOLO TRASCURABILE MA $\vec{d}$ DIPENDE DA SCELTA DI $\vec{r}_c$.

SCEGLIENDO $\vec{r}_c = \dfrac{\int d^3r\, \vec{r}\rho(\vec{r})}{\int d^3r\, \rho(\vec{r})} \longrightarrow \vec{d} = \vec{0}$

SE $Q=0$ DIPOLO DOMINANTE E $\vec{d}$ INDIPENDENTE DALLA SCELTA DI $\vec{r}_c$

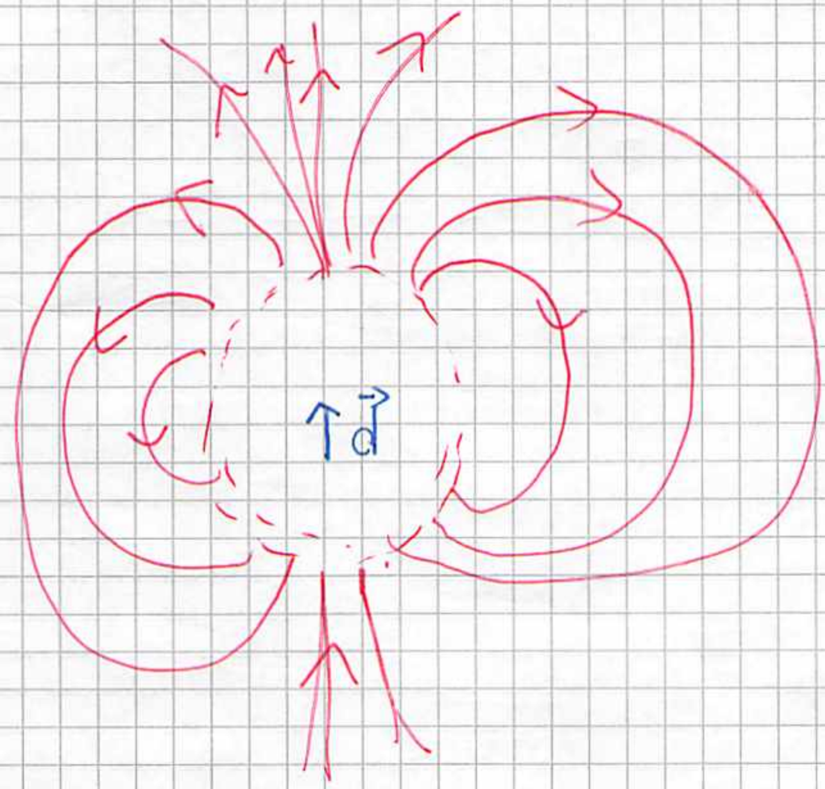
CAMPO ELETTRICO ASSOCIATO AD UN DIPOLO (SENZA MONOPOLO)

$$\vec{E}_d(\vec{r}) = -\frac{\partial}{\partial \vec{r}}\left[\frac{1}{4\pi\varepsilon_0}\frac{\vec{d}\cdot(\vec{r}-\vec{r}_c)}{|\vec{r}-\vec{r}_c|^3}\right] = -\frac{1}{4\pi\varepsilon_0}\frac{\partial}{\partial \vec{r}}\left[f(\vec{r})\,g(|\vec{r}-\vec{r}_c|)\right]$$

$$f(\vec{r}) = \vec{d}\cdot(\vec{r}-\vec{r}_c) \qquad g(|\vec{a}|) = \frac{1}{|a|^3}$$

$$\frac{\partial}{\partial \vec{r}}\left(\vec{d}\cdot(\vec{r}-\vec{r}_c)\right) = \vec{d} \qquad \frac{\partial g(|\vec{r}-\vec{r}_c|)}{\partial \vec{r}} = -3\frac{(\vec{r}-r_c)}{|\vec{r}-\vec{r}_c|^5}$$

$$\vec{E}_d(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{3\left[\vec{d}\cdot(\vec{r}-\vec{r}_c)\right](\vec{r}-\vec{r}_c) - \vec{d}\,|\vec{r}-r_c|^2}{|\vec{r}-\vec{r}_c|^5}$$



$\vec{E}_d$

ENERGIA PER PIAZZARE UNA DISTRIBUZIONE DI CARICA RIGIDA $\rho_1(\vec{r})$ IN UN CAMPO DI POTENZIALE $V_2(\vec{r})$ GENERATO DA UNA SECONDA DISTRIBUZIONE RIGIDA $\rho_2(\vec{r})$

$$V_2(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int d^3r' \, \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}$$

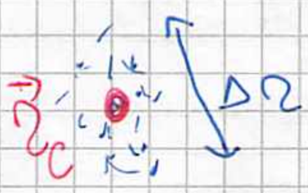
$$\text{ENER}_{\text{PIAZZAMENTO}} = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\Bigg\{ \overbrace{\int d^3r\, d^3r' \, \frac{[\rho_1(\vec{r})+\rho_2(\vec{r})][\rho_1(\vec{r}')+\rho_2(\vec{r}')]}{|\vec{r}-\vec{r}'|}}^{\text{ENERGIA PER CREARE } \rho_1+\rho_2}$$

$$- \underbrace{\int d^3r\, d^3r' \, \frac{\rho_1(\vec{r})\,\rho_1(\vec{r}')}{|\vec{r}-\vec{r}'|}}_{\text{ENER PER CREARE } \rho_1}$$

$$- \underbrace{\int d^3r\, d^3r' \, \frac{\rho_2(\vec{r})\,\rho_2(\vec{r}')}{|\vec{r}-\vec{r}'|}}_{\text{ENER PER CREARE } \rho_2} \Bigg\}$$

$$\text{ENER}_{\text{PIA.}} = \int d^3r \, \rho_1(\vec{r})\, V_2(\vec{r}) = \int d^3r \, \rho_2(\vec{r})\, V_1(\vec{r})$$

CASO CARICA $\rho_2(\vec{r})$ LOCALIZZATA



SE LA CARICA $V_2(\vec{r})$ VARIA POCO SULLA ZONA DI LOCALIZZAZIONE DI $\rho_1(\vec{r})$

$\Downarrow$

NELLA ZONA DI LOC. DI ρ_1

PRODOTTO SCALARE

GRADIENTE CAMPO ELETTRICO 2

$$V_2(\vec{r}) = V_2(\vec{r}_c) + (\vec{r} - \vec{r}_c) \cdot \frac{\partial V_2(\vec{r}_c)}{\partial \vec{r}} + O\left(|\vec{r} - \vec{r}_c| \frac{\partial \vec{E}_2}{\partial \vec{r}}\right)$$

$$\text{ENER}_{\text{PIA.}} = \int d^3r \, \rho_1(\vec{r}) V_2(\vec{r}) \simeq V_2(\vec{r}_c) \underbrace{\int d^3r \, \rho_1(\vec{r})}_{Q_1}$$

$$- \vec{E}_2(\vec{r}_c) \cdot \underbrace{\int d^3r \, (\vec{r} - \vec{r}_c) \rho_1(\vec{r})}_{\vec{d}_1}$$

$$\boxed{\text{ENE}_{\text{PIA}} = Q_1 V_2(\vec{r}_c) - \vec{d}_1 \cdot \vec{E}_2(\vec{r}_c)}$$

TERMINE SUCESSIVO $\propto$ QUADRUPOLO $\times$ GRADIENTE DEL CAMPO ELETTRICO

$\Downarrow$

SI MISURA IN NMR PER NUCLEI CON SPIN $> \frac{1}{2}$!

FORZA SU DISTRIBUZIONE RIGIDA $\rho_1(\vec{r})$

SE IL DIPOLO SI SPOSTA SENZA RUOTARE

$$\vec{F} = -\frac{\partial}{\partial \vec{r}_c}\left(\text{ENERGIA COLLOCAMENTO}\right) = -\frac{\partial}{\partial \vec{r}_c}\left[Q_1 V_2(\vec{r}_c)\right] + \frac{\partial}{\partial \vec{r}_c}\left[\vec{d}_1 \cdot \vec{E}_2(\vec{r}_c)\right]$$

$$= Q\,\vec{E}_2(\vec{r}_c) + d_{1x}\underbrace{\frac{\partial E_{2x}(\vec{r}_c)}{\partial \vec{r}_c}}_{\text{VETTORE}} + d_{1y}\underbrace{\frac{\partial E_{2y}(\vec{r}_c)}{\partial \vec{r}_c}}_{\text{VETTORE}}$$

$$+ \underbrace{d_{1z}\frac{\partial E_{2z}(\vec{r}_c)}{\partial \vec{r}_c}}_{\text{VETTORE}}$$

SE IL DIPOLO RUOTA PER ASSUMERE LA CONFIGURAZIONE DI ENERGIA MINIMA PER OGNI POSIZIONE $\vec{r}_c$ $\rightarrow$ $\vec{d}_1 \parallel \vec{E}_2(\vec{r})$ E STESSO VERSO

$$E_{min}(\vec{r}_c) = Q_1 V_2(\vec{r}_c) - d_1\, E_2(\vec{r}_c)$$

SE $Q_1 = 0$ PER MINIMIZZARE L'ENERGIA DIPOLO SI SPOSTA DOVE IL CAMPO È PIÙ INTENSO

$$\vec{F} = Q_1\,\vec{E}_2(\vec{r}) + d_1 \frac{\partial}{\partial \vec{r}_c} E_2(\vec{r}_c)$$

DIVERGENZA DI UN CAMPO VETTORIALE

$\vec{F}(\vec{r})$ = CAMPO VETTORIALE

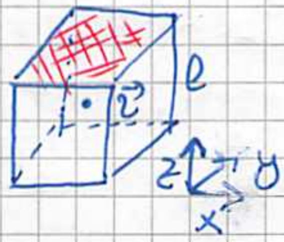
$\text{div}\,\vec{F}(\vec{r}) = f(\vec{r})$ CAMPO SCALARE

$$\text{div}\,\vec{F}(\vec{r}) \overset{def}{\equiv} \lim_{V\to 0} \Phi_S(\vec{F})\cdot\frac{1}{V}$$

DOVE S CONTIENE $\vec{r}$

V = VOLUME CONTENUTO IN S

DIVERGENZA IN COORDINATE CARTESIANE



V = CUBO CON CENTRO $\vec{r}$ E LATO ℓ

$\vec{r} = (x, y, z)$ $\quad V = \ell^3$

$$\Phi_{\text{FACCE}\perp z}(\vec{F}) = \int_{-\frac{\ell}{2}}^{\ell/2} dx' \int_{-\frac{\ell}{2}}^{\ell/2} dy' \left[F_z\left(x+x', y+y', z+\frac{\ell}{2}\right) - F_z\left(x+x', y+y', z-\frac{\ell}{2}\right)\right]$$

$$\overset{\ell\to 0}{=} \int_{-\frac{\ell}{2}}^{\ell/2} dx' \int_{-\ell/2}^{\ell/2} dy' \, \frac{\partial F_z}{\partial z}(x+x', y+y', z)\,\ell \overset{\ell\to 0}{=} \frac{\partial F_z(\vec{r})}{\partial z}\,\ell^3$$

$$\boxed{\text{div}\,\vec{F}(\vec{r}) = \lim_{\ell\to 0} \Phi_S(\vec{F})\,\frac{1}{\ell^3} = \frac{\partial F_x(\vec{r})}{\partial x} + \frac{\partial F_y(\vec{r})}{\partial y} + \frac{\partial F_z(\vec{r})}{\partial z}}$$

USANDO L'OPERATORE $\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$

$$\operatorname{div}\vec{F}(\vec{r}) = \vec{\nabla} \underset{\text{PRODOTTO SCALARE}}{\cdot} \vec{F}(\vec{r})$$

DIVERGENZA CAMPO ELETTRICO

$$\operatorname{div}\vec{E}(\vec{r}) = \lim_{V \to 0} \Phi_S(\vec{E}) \cdot \frac{1}{V} \overset{\text{GAUSS}}{=} \lim_{V \to 0} \frac{1}{\varepsilon_0} \frac{\text{CARICA IN } V}{V}$$

$$= \frac{1}{\varepsilon_0}\rho(\vec{r})$$

$$\boxed{\operatorname{div}\vec{E}(\vec{r}) = \frac{\rho(\vec{r})}{\varepsilon_0}}$$

PRIMA EQUAZIONE DI MAXWELL PER CAMPI STATICI

ESEMPIO:

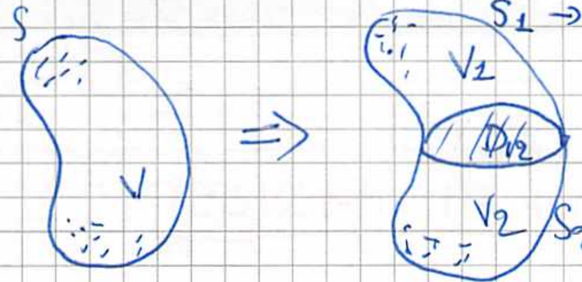
CAMPO ELETTRICO $\longleftrightarrow$ CARICA ASSOCIATA

$$\vec{E}(\vec{r}) = (ax+b)\hat{x} \qquad \rho(\vec{r}) = ?$$

$$\rho(\vec{r}) = \varepsilon_0 \operatorname{div}\vec{E}(\vec{r}) = \varepsilon_0\left(\frac{\partial E_x}{\partial x} + \overset{0}{\overset{\parallel}{\frac{\partial E_y}{\partial y}}} + \overset{0}{\overset{\parallel}{\frac{\partial E_z}{\partial z}}}\right) = \varepsilon_0\frac{\partial(ax+b)}{\partial x}$$

$$\rho(\vec{r}) = \varepsilon_0 a$$

TEOREMA DELLA DIVERGENZA

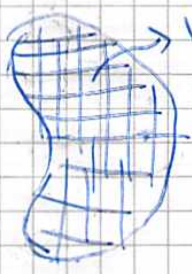


$S_1 \to$ CONTENENTE D_{12}

$S_2 \to$ CONTENENTE D_{12}

FLUSSO ATTRAVERSO D_{12} CONTRIBUISCE AI FLUSSI ATTRAVERSO S_1, S_2 IN MODO UGUALE IN MODULO MA OPPOSTO IN SEGNO

$$\Phi_S(\vec{F}) \overset{\uparrow}{=} \Phi_{S_1}(\vec{F}) + \Phi_{S_2}(\vec{F})$$



$\to$ TANTI VOLUMI PICCOLI V_i DI SUPERFICIE S_i CENTRATE IN $\vec{r}_i$

$$\Phi_S(\vec{F}) = \sum_i \frac{\Phi_{S_i}(\vec{F})}{V_i} V_i \overset{\lim_{V_i \to 0}}{=} \sum_i V_i \, div \vec{F}(\vec{r}_i)$$

$$\overset{V_i \to 0}{=} \int_V d^3 r \; div(\vec{F}(\vec{r}))$$

$$\boxed{\int_V d^3 r \; div \vec{F}(\vec{r}) = \Phi_S(\vec{F})}$$

TEOREMA DIVERGENZA

TEOREMA DI GAUSS

$$\Phi_S(\vec{E}) = \frac{\text{CARICA IN } S}{\varepsilon_0}$$

$\Downarrow$ DEFINIZIONE DIVERGENZA

$$\operatorname{div} \vec{E}(\vec{r}) = \frac{\rho(\vec{r})}{\varepsilon_0}$$

$\Downarrow$ TEOREMA DIVERGENZA

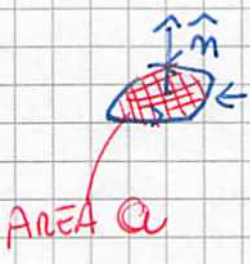
$$\Phi_S(\vec{E}) = \int_V d^3r \operatorname{div} \vec{E} = \int_V d^3r \frac{\rho(\vec{r})}{\varepsilon_0}$$

TEOREMA DI GAUSS

ROTAZIONALE DI UN CAMPO VETTORIALE

$$\text{rot}\,\vec{F}(\vec{r}) = \vec{G}(\vec{r}) = \text{CAMPO VETTORIALE}$$

$$\hat{n} \cdot \text{rot}\,\vec{F}(\vec{r})$$



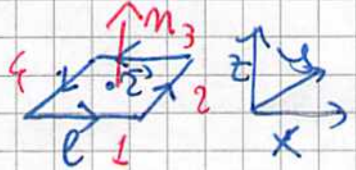
PICCOLO CIRCUITO CHIUSO C SU UNA SUPERFICIE PIANA $\perp$ A $\hat{n}$ ORIENTATA COME LA MANO DESTRA

- SENSO DI CIRCOLAZIONE DEL CAMMINO

$$\hat{n} \cdot \text{rot}\,\vec{F}(\vec{r}) \overset{def}{=} \lim_{a \to 0} \frac{\oint_{\mathcal{L}_C} \vec{F}(\vec{r}\,') \cdot d\vec{r}\,'}{a}$$

ROTAZIONALE IN COORDINATE CARTESIANE

COMPONENTE $z \Rightarrow \hat{n} = \hat{z}$



$$\oint \vec{F}(\vec{r}\,') \cdot d\vec{r}\,' = \overset{1}{\int_{-\frac{\ell}{2}}^{+\ell/2} dx' F_x\left(x+x', y-\frac{\ell}{2}, z\right)} + \overset{2}{\int_{-\ell/2}^{+\ell/2} dy' F_y\left(x+\frac{\ell}{2}, y+y', z\right)}$$

$$\overset{3}{- \int_{-\frac{\ell}{2}}^{\ell/2} dx' F_x\left(x+x', y+\frac{\ell}{2}, z\right)} - \int_{-\ell/2}^{\ell/2} dy' F_y\left(x-\frac{\ell}{2}, y+y', z\right)$$

$$\overset{\lim_{\ell\to 0}}{=} -\int_{-\ell/2}^{\ell/2} dx' \frac{\partial F_x}{\partial y}(x+x', y, z)\,\ell + \int_{-\ell/2}^{\ell/2} dy' \frac{\partial F_y}{\partial x}(x, y+y', z)\,\ell$$

$$\overset{\lim_{\ell\to 0}}{=} -\frac{\partial F_x}{\partial y}(\vec{r})\,\ell^2 + \frac{\partial F_y}{\partial x}(\vec{r})\,\ell^2$$

$$\boxed{\begin{aligned} \hat{z}\cdot \mathrm{rot}\,\vec{F}(\vec{r}) &= \frac{\partial F_y(\vec{r})}{\partial x} - \frac{\partial F_x(\vec{r})}{\partial y} \\ \hat{x}\cdot \mathrm{rot}\,\vec{F}(\vec{r}) &= \frac{\partial F_z(\vec{r})}{\partial y} - \frac{\partial F_y(\vec{r})}{\partial z} \\ \hat{y}\cdot \mathrm{rot}\,\vec{F}(\vec{r}) &= \frac{\partial F_x(\vec{r})}{\partial z} - \frac{\partial F_z(\vec{r})}{\partial x} \end{aligned}}$$

OTTENIBILI PER PERMUTAZIONE CICLICA $x \to y \to z$

USANDO L'OPERATORE $\vec{\nabla}$

PRODOTTO VETTORIALE

$$\mathrm{rot}\,\vec{F}(\vec{r}) = \vec{\nabla} \wedge \vec{F}(\vec{r})$$

ROTAZIONALE CAMPO ELETTRICO

$$\hat{n}\cdot \mathrm{rot}\,\vec{E}(\vec{r}) = \lim_{a\to 0} \underbrace{\oint_{\mathcal{L}_c} \vec{E}(\vec{r})\cdot d\vec{r}}_{=0}\, \frac{1}{a} = 0$$

$$\boxed{\mathrm{rot}\,\vec{E}(\vec{r}) = \vec{0}}$$

SECONDA EQUAZIONE DI MAXWELL STATICA

DERIVATE SECONDE

- $f(\vec{r})$ CAMPO SCALARE

$$\text{DIV}[\text{grad}\, f(\vec{r})] = \vec{\nabla}\cdot\vec{\nabla} f(\vec{r}) = \begin{matrix}\text{CAMPO}\\ \text{SCALARE}\end{matrix} =$$

$$= \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\cdot\left[\hat{x}\frac{\partial f}{\partial x} + \hat{y}\frac{\partial f}{\partial y} + \hat{z}\frac{\partial f}{\partial z}\right] =$$

$$= \frac{\partial^2}{\partial x^2} f(\vec{r}) + \frac{\partial^2}{\partial y^2} f(\vec{r}) + \frac{\partial^2}{\partial z^2} f(\vec{r}) = \nabla^2 f(\vec{r})$$

$$\boxed{\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}}$$ OPERATORE LAPLACIANO

- EQUAZIONE ONDE
  SCHRÖDINGER (MECCANICA QUANTISTICA)
  EQUAZIONE DIFFUSIONE (CALORE COMPOSIZIONE...)

POTENZIALE ELETTRICO (EQUAZIONE POISSON)

$$\vec{E}(\vec{r}) = -\vec{\nabla} V(\vec{r}) \qquad \vec{\nabla}\cdot(-\vec{\nabla} V(\vec{r})) = \vec{\nabla}\vec{E}(\vec{r}) =$$

$$= \frac{\rho(\vec{r})}{\varepsilon_0} \qquad \boxed{-\nabla^2 V(\vec{r}) = \frac{\rho(\vec{r})}{\varepsilon_0}}$$ EQUAZIONE POISSON PER POTENZIALE ELETTRICO

ALTRE DERIVATE SECONDE

- $f(\vec{r})$ CAMPO SCALARE

$$\mathrm{rot}\left[\mathrm{grad}\, f(\vec{r})\right] = \vec{\nabla} \wedge \vec{\nabla} f(\vec{r}) = \text{CAMPO VETTORIALE}$$

L'ORDINE DELLE DERIVATE MISTE NON CONTA

$$\left[\vec{\nabla} \wedge \vec{\nabla} f(\vec{r})\right]_x = \frac{\partial}{\partial y}\frac{\partial}{\partial z} f(\vec{r}) - \frac{\partial}{\partial z}\frac{\partial}{\partial y} f(\vec{r}) = 0$$

$$\boxed{\mathrm{rot}\left(\mathrm{grad}\, f(\vec{r})\right) = \vec{0} \quad \forall f(\vec{r})}$$

- $\vec{F}(\vec{r})$ CAMPO VETTORIALE

$$\mathrm{div}\left[\mathrm{rot}\, \vec{F}(\vec{r})\right] = \vec{\nabla} \cdot \left[\vec{\nabla} \wedge \vec{F}(\vec{r})\right] = \text{CAMPO SCALARE}$$

$$\frac{\partial}{\partial x}\left[\overset{+\bigcirc}{\frac{\partial}{\partial y} F_z(\vec{r})} - \overset{-\square}{\frac{\partial}{\partial z} F_y(\vec{r})}\right] + \frac{\partial}{\partial y}\left[\overset{+\triangle}{\frac{\partial}{\partial z} F_x(\vec{r})} - \overset{-\bigcirc}{\frac{\partial}{\partial x} F_z(\vec{r})}\right]$$

$$+ \frac{\partial}{\partial z}\left[\overset{+\square}{\frac{\partial}{\partial x} F_y(\vec{r})} - \overset{-\triangle}{\frac{\partial}{\partial y} F_x(\vec{r})}\right] = 0$$

$$\boxed{\mathrm{div}\left[\mathrm{rot}\, \vec{F}(\vec{r})\right] = 0 \quad \forall \vec{F}(\vec{r})}$$

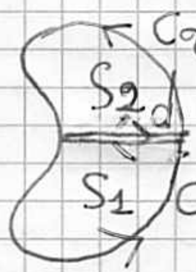
TEOREMA DI STOKES

PER SEMPLICITÀ ASSUMO UN CIRCUITO CHE GIACE IN UN PIANO (DIMOSTRAZIONE VALIDA ANCHE NEL CASO GENERALE)

SUPERFICIE S GIACE NEL PIANO

$\hat{n}$ NORMALE AL PIANO $\forall \vec{r} \in A$ S

$\vec{F}(\vec{r})$ = CAMPO VETTORIALE



IL CONTRIBUTO DI d UGUALE IN MODULO MA OPPOSTO IN SEGNO

$$\oint_{\mathcal{L}_C} \vec{F}(\vec{r}) \cdot d\vec{r} = \oint_{\mathcal{L}_{C_1}} \vec{F}(\vec{r}) \cdot d\vec{r} + \oint_{\mathcal{L}_{C_2}} \vec{F}(\vec{r}) \cdot d\vec{r}$$

$$= \sum_i a_i \oint_{\mathcal{L}_{C_1}} \vec{F}(\vec{r}) \cdot d\vec{r} \frac{1}{a_i} \overset{a_i \to 0}{=} \sum_i a_i \, \hat{n} \cdot \mathrm{rot}\, \vec{F}(\vec{r}_i)$$

$$= \int_S d^2\vec{a} \cdot \mathrm{rot}\, \vec{F}(\vec{r})$$

$$\boxed{\oint_{\mathcal{L}_C} \vec{F}(\vec{r}) \cdot d\vec{r} = \int_S d^2\vec{a} \cdot \mathrm{rot}\, \vec{F}(\vec{r})}$$

TEOREMA DI STOKES

DIMOSTRATO QUANDO C E S $\in$ PIANO MA GENERALIZZABILE

ESEMPIO GENERALIZZAZIONE

$C \in$ PIANO $\quad S \notin$ PIANO $\quad \hat{n}(\vec{r})\ \vec{r} \in S$

$S_1 \in$ PIANO $\hat{n}_1 \perp$ PIANO UNIFORME

DIMOSTRATO PAGINA PRECEDENTE

$$\oint_{\mathcal{L}_C} \vec{F}(\vec{r})\cdot d\vec{r} = \int_{S_1} d^2\vec{a}\cdot(\mathrm{rot}\,\vec{F}(\vec{r})) = \int_S d^2\vec{a}\cdot(\mathrm{rot}\,\vec{F}(\vec{r}))$$

$$\underbrace{-\int_S d^2\vec{a}\cdot(\mathrm{rot}\,\vec{F}(\vec{r})) + \int_{S_1} d^2\vec{a}\cdot(\mathrm{rot}\,\vec{F}(\vec{r}))} =$$

$\hat{n}_1$ ENTRANTE

$\hat{n}$ uscente volume

TEOREMA DIVERGENZA

$$-\oint d^2\vec{a}\cdot(\mathrm{rot}\,\vec{F}(\vec{r})) = \int_V d^3r\ \underbrace{\mathrm{div}(\mathrm{rot}\,\vec{F}(\vec{r}))}_{=0} = 0$$

SUP CHIUSA CONTENENTE IL VOLUME FRA S_1 E $S = V$

$$\boxed{\oint_{\mathcal{L}_C} \vec{F}(\vec{r})\cdot d\vec{r} = \int_S d^2\vec{a}\cdot(\mathrm{rot}\,\vec{F}(\vec{r})) \qquad S \notin \text{PIANO}}$$

LEMMA SE $\mathrm{rot}(\vec{F}(\vec{r})) = \vec{0}$ IN TUTTO LO SPAZIO

$\oint_{\mathcal{L}_C} \vec{F}(\vec{r})\cdot d\vec{r} = 0 \Rightarrow$ CAMPO CONSERVATIVO

DIMOSTRAZIONE FORZE CENTRALI CONSERVATIVE (BIS)

$$\vec{F}(\vec{r}) = g(r)\,\vec{r} = g\left(\sqrt{x^2+y^2+z^2}\right)\left(\hat{x}x + \hat{y}y + \hat{z}z\right)$$

$$\hat{z} \cdot \mathrm{rot}\,\vec{F}(\vec{r}) = \frac{\partial}{\partial x}\left(g(r)\,y\right) - \frac{\partial}{\partial y}\left(g(r)\,x\right) =$$

$$= \frac{x}{r} g'(r)\,y - \frac{y}{r} g'(r)\,x = 0$$

$\Downarrow$ ANALOGAMENTE PER COMPONENTI y E z

$\mathrm{rot}\,\vec{F}(\vec{r}) = 0 \Rightarrow$ POTENZIALE CONSERVATIVO

LEGGE DI COULOMB

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int d^3r' \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|^2}\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|} + E_\infty$$

$$\Updownarrow$$

$$\begin{cases} \vec{\nabla}\cdot\vec{E}(\vec{r}) = \dfrac{\rho(\vec{r})}{\varepsilon_0} \\ \vec{\nabla}\wedge\vec{E}(\vec{r}) = \vec{0} \end{cases}$$

EQUAZIONI DI MAXWELL STATICHE

TEOREMA DIVERGENZA

TEOREMA STOKES

DEF DIV

DEF ROT

$$\Updownarrow$$

$$\begin{cases} \nabla^2 V(\vec{r}) = -\dfrac{\rho(\vec{r})}{\varepsilon_0} \\ \vec{E}(\vec{r}) = -\vec{\nabla}V(\vec{r}) \end{cases}$$

EQUAZIONE POISSON

$$\Updownarrow$$

$$\begin{cases} \Phi_S(\vec{E}) = \dfrac{Q_{IN\ S}}{\varepsilon_0} \\ \oint_{\mathcal{L}_C} \vec{E}(\vec{r})\cdot dr = 0 \end{cases}$$

EQUAZIONE GAUSS

EQUAZIONE CIRCUITAZIONE

ENERGIA PER FORMARE UNA DISTRIBUZIONE

IDENTITÀ MATEMATICA

$$\vec{\nabla}(\vec{E}(\vec{r})\cdot V(\vec{r})) = \frac{\partial}{\partial x}[E_x(\vec{r})V(\vec{r})] + \frac{\partial}{\partial y}[E_y(\vec{r})V(\vec{r})] + \frac{\partial}{\partial z}[E_z(\vec{r})V(\vec{r})]$$

$$= V(\vec{r})\cdot[\operatorname{div}\vec{E}(\vec{r})] - |\vec{E}(\vec{r})|^2 = V(\vec{r})\frac{\rho(\vec{r})}{\varepsilon_0} - E(r)^2$$

$$V(\vec{r})\rho(\vec{r}) = \varepsilon_0[E(\vec{r})]^2 + \varepsilon_0\vec{\nabla}\cdot(\vec{E}(\vec{r})\cdot V(\vec{r}))$$

$$U = \frac{1}{2}\int d^3r\, V(\vec{r})\rho(\vec{r}) = \frac{1}{2}\varepsilon_0\int d^3r[E(r)]^2$$

$$+\frac{1}{2}\varepsilon_0\int d^3r\, \operatorname{div}[\vec{E}(\vec{r})V(\vec{r})]$$

TEOREMA DIVERGENZA

$$\frac{1}{2}\varepsilon_0\int_S d^2\vec{a}\cdot\vec{E}(\vec{r})\,V(\vec{r}) = 0\ !$$

SE CARICA LOCALIZZATA R RAGGIO S

$$\underbrace{R^2 \quad \frac{1}{R^2} \quad \frac{1}{R}}_{} \quad \propto \frac{1}{R} \to 0 \quad R\to\infty$$

$$\boxed{U = \frac{1}{2}\varepsilon_0\int d^3r[E(r)]^2}$$